Abstract

Hajime Kaneko (Kyoto Univ.)
Title 1: Algebraic independence of real numbers with low density of nonzero digits
Abstract: In this talk we will give new criteria for algebraic independence of the numbers $\xi = \sum_{n=1}^{\infty} \alpha^{-w(n)}$, where $\alpha \geq 2$ is an integer and $(w(n))_{n=1}^{\infty}$ is a strictly increasing sequence of nonnegative integers. Applying our criteria, we deduce the algebraic independence of such ξ in the case of $\lim_{n\to\infty} w(n+1)/w(n) = 1$, which was impossible by early methods.

Title 2: On the complexity of the binary expansions of algebraic irrational numbers
Abstract: Borel conjectured that all algebraic irrational numbers ξ are normal in base 2. Namely, all finite words with letters from the alphabet $\{0, 1\}$ occur with the proper frequency. However, it is still unknown whether the word 11 appears infinitely often in the binary expansions of $\sqrt{2}$. We derive new, improved lower bounds of the number of digit changes in the binary expansions of algebraic irrational numbers.

Hacène Belbachir (Université des Science et de la Technologie Houari Boumedienne, Algeria)
Title: Preserving log-concavity and generalized triangles
Abstract: A sequence of positive numbers $\{x_k\}_k$ is log-concave if for each integer $k > 0$, we have $x_{k-1}x_{k+1} \leq x_k^2$. Several classical sequences are log-concave as Stirling numbers of the first and second kind, Euler numbers and all sequence of binomial coefficients located in a ray or a transversal of the Pascal triangle $\{\binom{n+id}{k+i\sigma}_i\}$.

Let $\{a(n,k)\}_{0\leq k\leq n}$ be a triangle of positive numbers and consider the following linear transformation

$$z_n = \sum_{k=0}^{n} a(n,k)x_k, \quad (n = 0, 1, 2, \dots) \tag{1}$$

We say that the linear transformation (1) preserves log-concavity if the log-concavity of $\{x_n\}$ implies that of $\{z_n\}$. It is, for example, the case for triangles $a(n,k) = \binom{a+n}{b+k}$ and $a(n,k) = \begin{bmatrix} n \\ k \end{bmatrix} q^{k(k-n)}$ ($\begin{bmatrix} n \\ k \end{bmatrix}$ be the q-binomial coefficient).

Our talk is about the preservation of log-concavity for p-triangles: let p a positive integer, a p-triangle is a bi-indexed sequence of positive numbers $\{a_p(n,k)\}_{0\leq k\leq np}$ satisfying $a_p(n,k) = 0$ for $k < 0$ or $k > np$. We illustrate a 4-triangle as follows

$n\backslash k$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	$\star$																
2	$\star$	$\star$	$\star$	$\star$	$\star$												
3	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$								
4	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$				
5	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$

As example of p-triangles, let us consider the ordinary multinomials: for $q \geq 1$ and $L \geq 0$ two integers, and $k = 0, 1, \ldots, qL$, the ordinary multinomial coefficient $\binom{L}{k}_q$ is defined as the k-th coefficient in the development

$$(1 + x + x^2 + \cdots + x^q)^L = \sum_{k \geq 0} \binom{L}{k}_q x^k.$$

Considering a p-triangle of positive numbers and a log-concave sequence $\{x_n\}_n$, we shall specify the situations for witch we obtain a log-concave sequence $\{z_n\}_n$ using the following linear transformation

$$z_n = \sum_{k=0}^{pn} a_p(n,k) x_k.$$

Georges Grekos (Univ. Saint-Etienne, France)
Title: Open problems on densities

Abstract: A density on subsets of $\mathbf{N}_0 := \{0, 1, 2, \ldots\}$ is a function δ, defined on the power set $\mathcal{P}(\mathbf{N}_0)$, taking values in $[0, 1]$, such that $\delta(A) \leq \delta(B)$ if $A \subseteq B$. This definition is usually accompagnied by the conditions $\delta(\mathbf{N}_0) = 1$ and $\delta(F) = 0$ for any finite set F of non-negative integers. A density δ may be defined only on *certain* subsets of $\mathbf{N}_0$.

In this talk I shall mostly consider *asymptotic* densities: for instance, the upper and the lower limits, as N tends to $+\infty$, of

$$\Big(\sum_{n \leq N} n^{\alpha}\Big)^{-1} \Big(\sum_{n \in A, n \leq N} n^{\alpha}\Big)$$

(α fixed ≥ -1 ; $\alpha = -1$: *logarithmic* density ; $\alpha = 0$: the usual or "natural" asymptotic density), or of

$$\frac{\log |A \cap [1, N]|}{\log N}$$

(*exponential* density), or the upper and lower *Banach* (or *uniform*) densities.

I shall present and comment:

- Problems concerning the definitions (especially the dependence of densities on parameters).
- Problems concerning the density set. If A is a fixed subset of $\mathbf{N}_0$, its *density set* is defined as

$$S(A) := \{(\overline{\delta}(B), \underline{\delta}(B)) \in [0, 1]^2 \ ; \ B \subseteq A\}.$$

- Problems in Number Theory where the concept of density is crucial. An example is the following problem of D.A. Klarner: Les S be a subset of $\mathbf{N}_0$ such that $0 \in S$, S is closed

under the applications $\alpha_1(x) = 2x$, $\alpha_2(x) = 3x + 2$, and $\alpha_3(x) = 6x + 3$, and S is *minimal.* Is it true that the natural density of S is zero?

Shin-ichi Yasutomi (Suzuka National College of Technology)
Jun-ichi Tamura

Title: Lagarange type theorems on formal power series

Abstract: It is known that Lagrange's Theorems is not always true in the sense that there exist some quadratic elements $\alpha \in \mathbb{Q}((t^{-1}))$ over $\mathbb{Q}(t)$ having nonperiodic continued fraction expansion. We present an information@related to the continued fraction expansion for quadratic elements in $\mathbb{Q}((t^{-1}))$. We also present a two dimensional version of our information related to AJPA (algebraic Jacobi-Perron algorithm) on formal power series.

Ryotaro Okazaki (Doshisha Univ.)

Title: Weber's Class Number Problem by Diophantine Method

Abstract: Let $K_n = \mathbf{Q}(\cos(2\pi/2^{n+2}))$. Denote by h_n its ideal class number. Weber calculated $h_1 = h_2 = h_3$ and asked whether $h_n = 1$ for every $n \geq 1$. Later Bauer and Masley showed $h_4 = 1$ and Linden showed $h_5 = 1$. Recently K. Horie initiated a project of proving this conjecture of Weber's. Fukuda and Komatsu made a further progress.

Their method is based on the identity between the ideal class number and the index of cyclotomic units. A general idea is that a lower bound for units leads to an upper bound for the class number. The mentioned authors investigated the structure of units in more detail so that they obained a method for handling each possible prime divisor. However, they still use lower bounds on units.

In this talk, we will give lower bounds for units in K_n to K_{n-1} equals ± 1. Then, we will discuss their application in Weber's class number problem. Amazingly, the arguments are more Diophantine than algebraic or analytic.

In the same conference, Morisawa will talk about generalization to other cyclotomic fields.

Takayuki Morisawa (Waseda Univ.)

Title: Mahler Measure of Horie Unit and Weber's Class Number Problem in the Cyclotomic Z_p-extension of Q

Abstract: Let p be a prime number. It is an interesting problem to consider whether a prime number l divides the class numbers of the intermediate fields of the cyclotomic Z_p-extension of Q. We call this problem 'Weber's Class Number Problem'. In the case $p = 2$, R. Okazaki developed a theory for this problem by using Mahler measure. In this conference, we talk about the case where p is an odd prime number.

Takafumi Miyazaki (Tokyo Metropolitan Univ.)
Title: Generalizations of classical results on Jesmanowicz' conjecture concerning Pythagorean triples
Abstract: Let a, b, c be relatively prime positive integers such that $a^2 + b^2 = c^2$ with even b. In 1956, Jeśmanowicz conjectured that the equation $a^x + b^y = c^z$ has only the positive integral solution $(x, y, z) = (2, 2, 2)$. There are some classical and celebrated results on this conjecture. In particular, due to Sierpiński, Jeśmanowicz, Lu, Ko, Podsypanin and Dem'janenko. In this talk, we broadly generalize them by proving the conjecture in the cases where $a \equiv \pm 1 \pmod{b}$ or $c \equiv 1 \pmod{b}$. Our methods are completely elementary.

Yusutsugu Fujita (Nihon Univ.)
Title: Generators for the elliptic curves $y^2 = x^3 - nx$
Abstract: Let $E : y^2 = x^3 - nx$ be an elliptic curve over the rationals with a positive integer n. Mordell's theorem asserts that the group of rational points on E is finitely generated. Our interest is in the generators for its free part. Duquesne (2007) showed that if $n = (2k^2 - 2k + 1)(18k^2 + 30k + 17)$ is square-free, then certain two points of infinite order can always be in a system of generators. We generalize this result and show that the same is true for "infinitely many" infinite families $n = n(k, l)$ with two variables.

This is the joint work with Nobuhiro Terai (Division of General Education, Ashikaga Institute of Technology).

Laszlo Szalay (Univ. of West Hungary)
Title: Diophantine triples and binary recurrences

Abstract: A Diophantine n-tuple is a set $S = \{a_1, a_2, \ldots, a_n\}$ of positive integers such that the product of any two elements of S is one less than a perfect square. In the third century, Diophantus of Alexandria started to investigate such sets, and he found the rational quadruple $\{1/16, 33/16, 17/4, 105/16\}$. For integers it is conjectured that there is no Diophantine quintuple.

There exist several variants of the problem. Consider now that case when we replace the square property by elements of a given binary recurrence. More precisely, suppose that $\{G_n\}_{n=0}^{\infty}$ is a binary recurrence sequence of integers given by the initial values G_0 and G_1, and by the recurrence relation

$$G_n = AG_{n-1} + BG_{n-2}, \quad (n \geq 2).$$

Now, we would like to study the system of three diophantine equations

$$ab + 1 = G_x, \quad ac + 1 = G_y, \quad bc + 1 = G_z \tag{2}$$

in non-negative integers a, b, c and x, y, z.

In general, we showed that, apart from some particular classes, the system (2) possesses only finitely many solution if $D = A^2 + 4B > 0$ and the sequence $\{G_n\}$ is non-degenerate. Further, in case of the Fibonacci and the Lucas sequence we could completely solve (2) by applying different technique. The results above are joint work with F. Luca and, in part, with C. Fuchs.

Daisuke Shiomi (Nagoya Univ.)

A congruence modulo p of zeta polynomial for cyclotomic funtion fields

Abstract: As an analouge of cyclotomic field, the m-th cyclotomic function field K_m is defined as adding m-th torsion points of Carlitz module to the rational function field of characteristic p. Let h_m^- be the relative class number of K_m. In 1999, L. Guo and L. Shu showed the following congruence relation:

$$h_{P^n}^- \equiv h_P^- \mod p,$$

where P is a monic irreducible polynomial and n is an integer.

In this talk, we will generalize the above congruence from the view point of congruence zeta function.

Makoto Nagata (Osaka Univ. Pharm.)

Title: Siegel's lemma and linear spaces of random variables

Abstract: The original Siegel's lemma is an estimation of the sizes of integral solutions of a system of linear equations. With respect to the system of linear equations, we consider the *other* system of linear equations in a linear space of random variables. Let Y_ϵ be a solution of the latter. (Or it would be more accurate to say that we consider the other system of linear maps. Let Y_ϵ be an element in its kernel.) Our main results are the following.

(1) With the size of the solution Y_ϵ, *our* Siegel's lemma is presented.

(2) For the trivial solution $Y_\epsilon = 0$, our Siegel's lemma corresponds to the classical one, i.e., Siegel's lemma without the height of the linear subspace.

(3) In the case of the linear subspace (in Siegel's lemma) defined on the rationals, for a nontrivial solution Y_ϵ, our Siegel's lemma corresponds to the modern one, i.e., Siegel's lemma with the height of the linear subspace.

The *principle* underlying our Siegel's lemma is neither the pigeonhole principle nor Minkowski's theorem for successive minima in the geometry of numbers, but a property of orthogonal vectors on a sphere.

Masanori Katsurada (Keio Univ.)

Title: Asymptotic expansions for certain multiple q-integrals and q-differentials of Thomae-Jackson type

Abstract: Let q be a real parameter with $0 < q < 1$, and $\varphi(u)$ a function continuous on the interval $[0, x]$. A q-analogue of the ordinary integral $\int_0^x \varphi(u)du$ is denoted by $\int_0^x \varphi(u)d_qu$, which was first formulated by Thomae in 1869 and extensively studied by Jackson during 1910–1951. It is known that $\int_0^x \varphi(u)d_qu$ tends to $\int_0^x \varphi(u)du$ when $q \to 1-0$. A q-analogue of the ordinary differentiation was also formulated so as that it tends to the original differentiation when $q \to 1-0$. We shall show in this talk complete asymptotic expansions exist when $q \to 1-0$ for certain (weighted) multiple q-integrals and q-differentials under fairly generic situations. Several applications of these expansions will be further presented.

Takumi Noda (Nihon Univ.)
Title: On generalized Lipschitz-type formulae and applications
Abstract: Let $s = \sigma + it$ be a complex variable, and let $\mathscr{H}^+ = \{z \in \mathbb{C} | 0 < \arg(z) < \pi\}$ and $\mathscr{H}^- = \{z \in \mathbb{C} | -\pi < \arg(z) < 0\}$ be the complex half planes. The classical Lipschitz formula asserts that

$$\sum_{m=-\infty}^{\infty} (z+m)^{-k} = \frac{(-2\pi i)^k}{(k-1)!} \sum_{n=1}^{\infty} n^{k-1} \exp(2\pi i n z) \quad (z \in \mathscr{H}^+),$$

for any positive integer k. In studying automorphic L-functions and the spectral theory of zeta-functions, fundamental tools are supplied from some analytic properties of (holomorphic and non-holomorphic) Eisenstein series, among which their Fourier series expansions play crucial roles. The Lipschitz formula above is a key ingredient for obtaining such Fourier series expansions.

The purpose of this talk is to present a certain Lipschitz-type formula and its generalization via Mellin-Barnes type integrals. We further introduce a class of double Eisenstein series for $SL(2, \mathbb{Z})$ defined on $(\mathscr{H}^+)^2 \cup (\mathscr{H}^-)^2$, and show their transformation properties by using the generalized Lipschitz-type formulae.