On the irrationality of a certain series

Florian Luca

Let $q > 1$ be an integer. In 1995, Daniel Duverney used elementary considerations together with results concerning the distribution of numbers which are sums of two squares to show that the Tchakaloff number

$$\sum_{n\geq 1} \frac{1}{q^{n(n+1)/2}}$$

is not quadratic. In my talk, I will extend Duverney's method to show that if $K > 1$ is any given constant and $f(X) \in \mathbf{Q}(X)$ is any integer valued quadratic polynomial with positive leading term, then the number

$$\sum_{n\geq 1} \frac{a_n}{q^{f(n)}} \qquad \text{with } 0 < |a_n| < K \text{ for all } n \geq 1$$

is not quadratic. The proof uses sieves and a result of Iwaniecz on primes represented as a sum of two values of a quadratic polynomial.