Some metric results in algebraic independence theory

Yu. Nesterenko, Moscow State University

It is well known that for any point $\overline{\omega} = (\omega_1, \ldots, \omega_m) \in \mathbf{R}^m$ there exists an infinite sequence of distinct polynomials $P \in \mathbf{Z}[x_1, \ldots, x_m]$ satisfying

$$0 < |P(\overline{\omega})| < e^{-c_1(\deg P + \log H(P))^{m+1}},$$

where $H(P)$ is the hight of P and c_1 is a positive constant depending only on m. On the other side it was conjectured in 1974 that for almost every point $\overline{\omega}$ one can find a constant $c_2 = c_2(m, \overline{\omega}) > 0$ such that for any $P \neq 0$ we have

$$|P(\overline{\omega})| > e^{-c_2(\deg P + \log H(P))^{m+1}}.$$

This conjecture was proved in 1974 in case $m = 1$ and in general case the weaker result with the exponent $m + 2$ instead of $m + 1$ was proved. There exist p-adic and complex versions of this conjecture. In 1994 the complex version was proved by F.Amoroso. In October 2006 S.Michailov succeed to prove the real case. This proof can be extended to p-adic and complex cases. Discussion of the proof will be the main subject of the talk.

Measure of irrationality for $\log 3$ (after Salikhov)

Yu. Nesterenko, Moscow State University

In 1985 G.Rhin proved that the measure of irrationality for $\log 3$ can be bounded as $\mu(\log 3) \leq 8.616$. The better result $\mu(\log 3) \leq 5.125$ was proved very recently by V.Salikhov who used ideas of M.Hata but a different integral construction.

Theorem. *Let be $q, p_1, p_2 \in \mathbf{Z}$, $Q = \max(|q|, |p_1|, |p_2|)$. Then*

$$|q + p_1 \log 2 + p_2 \log 3| \geq Q^{-4.125},$$

if Q is sufficiently large.

The proof of this theorem will be discussed.