

# はじめに (数学基礎 B2)

数学基礎 B = 線形代数

教科書 「要点明解 線形数学」 培風館

(第1章 行列)

(第2章 連立1次方程式)

- ▶ 第3章 行列式
- ▶ 第4章 行列の対角化

講義の情報 <http://mathweb.sc.niigata-u.ac.jp/~hoshi/teaching-j.html>

シラバス [LINK](#)

- ▶ ノートを取りながら講義を聴くこと。  
(ノートを回収して確認する可能性があります)
- ▶ 講義 → 小テスト (理解度確認テスト, 学務情報システム内)

### 定理 3.6

$n$  次正方行列  $A$  のある行を  $k$  倍した行列  $A''$  に対して,  $|A''| = k|A|$ .

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$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, A'' = \begin{pmatrix} ka & kb \\ c & d \end{pmatrix}$$

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$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, A'' = \begin{pmatrix} ka & kb \\ c & d \end{pmatrix} \Rightarrow |A''| = kad - kbc = k(ad - bc) = k|A|.$$

### 定理 3.7

$n$  次正方行列  $A$  のある行が行ベクトルの和  $\mathbf{a} + \mathbf{a}'$

$$\Rightarrow |A| = \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \mathbf{a} + \mathbf{a}' \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \mathbf{a} \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} + \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \mathbf{a}' \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} .$$

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$$A = \begin{pmatrix} \mathbf{a} + \mathbf{a}' & \mathbf{b} + \mathbf{b}' \\ c & d \end{pmatrix} \Rightarrow$$

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$$A = \begin{pmatrix} \mathbf{a} + \mathbf{a}' & \mathbf{b} + \mathbf{b}' \\ c & d \end{pmatrix} \Rightarrow |A| = (a + a')d - (b + b')c$$

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$$= (ad - bc) + (a'd - b'c) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} a' & b' \\ c & d \end{vmatrix}.$$



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$n$  次正方行列  $A$  のある  $(i)$  行に他の行  $(j)$  行の  $k$  倍を加えた行列  $A'''$  に対して,  $|A'''| = |A|$ .

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$$\therefore |A'''| = \begin{vmatrix} a_1 \\ a_i + k a_j \\ a_j \\ a_n \end{vmatrix}$$

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$$\stackrel{3.5}{=} |A| + k \cdot 0 = |A|.$$

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$$A \rightarrow \cdots \rightarrow \left( \begin{array}{cccc} a & * & \cdots & * \\ 0 & \color{red}{\boxed{B}} & & \\ \vdots & & & \\ 0 & & & \end{array} \right) \text{の形にできれば, (定理 3.3 より)}$$

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$|A|$  を  $a \cdot |B|$  を用いて計算できる.



# 例

$$\left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right|$$

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3.3

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$$\begin{array}{c} \left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \updownarrow \\ \text{I} \end{array} \\ \text{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \downarrow \times (-3) \\ \text{III} \end{array} \\ \text{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \downarrow \times 1 \\ \text{III} \end{array} \\ \text{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right| \end{array}$$

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$$\stackrel{3.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

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# 例

$$\left| \begin{array}{cccc|c} 1 & 2 & 3 & -1 & \\ -3 & 2 & 7 & 11 & \\ 0 & 5 & 9 & 16 & \\ -2 & 0 & 1 & 6 & \end{array} \right|$$

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$$\begin{vmatrix} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\text{I}} = - \begin{vmatrix} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\times(-3)} \begin{vmatrix} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\times 1} \begin{vmatrix} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{vmatrix}$$

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$$\stackrel{3.3}{=} - \begin{vmatrix} 8 & 4 \\ -1 & 0 \end{vmatrix} = (-1) \cdot 4 = -4.$$

# 例

$$\begin{vmatrix} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{vmatrix} \xrightarrow{\times 3} \begin{vmatrix} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{vmatrix} \xrightarrow{\times 2}$$

# 例

$$\left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \stackrel{\boxed{\text{I}}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \times (-3) \stackrel{\boxed{\text{III}}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \times 1 \stackrel{\boxed{\text{III}}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right|$$

$$\stackrel{3.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

# 例

$$\left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \stackrel{\times 3}{=} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \stackrel{\times 2}{=} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right|$$

# 例

$$\left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \stackrel{\boxed{\text{I}}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \times (-3) \stackrel{\boxed{\text{III}}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \times 1 \stackrel{\boxed{\text{III}}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right|$$

$$\stackrel{3.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

# 例

$$\left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \stackrel{\times 3}{=} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \stackrel{\times 2}{=} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right|$$

# 例

$$\begin{vmatrix} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\text{I}} = - \begin{vmatrix} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\times(-3)} \begin{vmatrix} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\text{III}} = - \begin{vmatrix} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{vmatrix} \xrightarrow{\times 1}$$

$$\stackrel{3.3}{=} - \begin{vmatrix} 8 & 4 \\ -1 & 0 \end{vmatrix} = (-1) \cdot 4 = -4.$$

# 例

$$\begin{vmatrix} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{vmatrix} \xrightarrow{\times 3} \begin{vmatrix} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{vmatrix} \xrightarrow{\text{III}} \begin{vmatrix} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{vmatrix} \stackrel{3.3}{=}$$

# 例

$$\left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \stackrel{\text{I}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\times(-3)} \stackrel{\text{III}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\times 1} \stackrel{\text{III}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right|$$

$$\stackrel{3.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

# 例

$$\left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\times 3} \stackrel{\text{III}}{=} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\times 2} \stackrel{\text{III}}{=} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \stackrel{3.3}{=} \left| \begin{array}{ccc} 8 & 16 & 8 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right|$$

# 例

$$\left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \stackrel{\boxed{\text{I}}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \stackrel{\times(-3)}{\downarrow} \stackrel{\boxed{\text{III}}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \stackrel{\times 1}{\downarrow} \stackrel{\boxed{\text{III}}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right|$$

$$\stackrel{3.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

# 例

$$\left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \stackrel{\times 3}{\downarrow} \stackrel{\boxed{\text{III}}}{=} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \stackrel{\times 2}{\downarrow} \stackrel{\boxed{\text{III}}}{=} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \stackrel{3.3}{=} \left| \begin{array}{ccc} 8 & 16 & 8 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right|$$

$$\boxed{\text{II}} \\ =$$

# 例

$$\begin{array}{c} \boxed{\text{I}} \\ \left| \begin{array}{ccc|c} 3 & 2 & 1 & \\ 1 & -2 & -1 & \\ -1 & 1 & 1 & \end{array} \right| \begin{array}{l} \updownarrow \\ = - \end{array} \left| \begin{array}{ccc|c} 1 & -2 & -1 & \\ 3 & 2 & 1 & \\ -1 & 1 & 1 & \end{array} \right| \begin{array}{l} \downarrow \times (-3) \\ = - \end{array} \left| \begin{array}{ccc|c} 1 & -2 & -1 & \\ 0 & 8 & 4 & \\ -1 & 1 & 1 & \end{array} \right| \begin{array}{l} \downarrow \times 1 \\ = - \end{array} \left| \begin{array}{ccc|c} 1 & -2 & -1 & \\ 0 & 8 & 4 & \\ 0 & -1 & 0 & \end{array} \right| \end{array}$$

$$\stackrel{3.3}{=} - \left| \begin{array}{cc|c} 8 & 4 & \\ -1 & 0 & \end{array} \right| = (-1) \cdot 4 = -4.$$

# 例

$$\begin{array}{c} \boxed{\text{III}} \\ \left| \begin{array}{cccc|c} 1 & 2 & 3 & -1 & \\ -3 & 2 & 7 & 11 & \\ 0 & 5 & 9 & 16 & \\ -2 & 0 & 1 & 6 & \end{array} \right| \begin{array}{l} \downarrow \times 3 \\ = \end{array} \left| \begin{array}{cccc|c} 1 & 2 & 3 & -1 & \\ 0 & 8 & 16 & 8 & \\ 0 & 5 & 9 & 16 & \\ -2 & 0 & 1 & 6 & \end{array} \right| \begin{array}{l} \downarrow \times 2 \\ = \end{array} \left| \begin{array}{cccc|c} 1 & 2 & 3 & -1 & \\ 0 & 8 & 16 & 8 & \\ 0 & 5 & 9 & 16 & \\ 0 & 4 & 7 & 4 & \end{array} \right| \stackrel{3.3}{=} \left| \begin{array}{ccc|c} 8 & 16 & 8 & \\ 5 & 9 & 16 & \\ 4 & 7 & 4 & \end{array} \right| \end{array}$$

$$\boxed{\text{II}} = 8 \cdot \left| \begin{array}{ccc|c} 1 & 2 & 1 & \\ 5 & 9 & 16 & \\ 4 & 7 & 4 & \end{array} \right|$$



# 例

$$\begin{array}{c} \boxed{\text{I}} \\ \left| \begin{array}{ccc|c} 3 & 2 & 1 & \\ 1 & -2 & -1 & \\ -1 & 1 & 1 & \end{array} \right| \begin{array}{l} \updownarrow \\ = - \end{array} \left| \begin{array}{ccc|c} 1 & -2 & -1 & \\ 3 & 2 & 1 & \\ -1 & 1 & 1 & \end{array} \right| \begin{array}{l} \downarrow \times (-3) \\ = - \end{array} \boxed{\text{III}} \left| \begin{array}{ccc|c} 1 & -2 & -1 & \\ 0 & 8 & 4 & \\ -1 & 1 & 1 & \end{array} \right| \begin{array}{l} \downarrow \times 1 \\ = - \end{array} \boxed{\text{III}} \left| \begin{array}{ccc|c} 1 & -2 & -1 & \\ 0 & 8 & 4 & \\ 0 & -1 & 0 & \end{array} \right| \end{array}$$

$$\stackrel{3.3}{=} - \left| \begin{array}{cc|c} 8 & 4 & \\ -1 & 0 & \end{array} \right| = (-1) \cdot 4 = -4.$$

# 例

$$\begin{array}{c} \boxed{\text{III}} \\ \left| \begin{array}{cccc|c} 1 & 2 & 3 & -1 & \\ -3 & 2 & 7 & 11 & \\ 0 & 5 & 9 & 16 & \\ -2 & 0 & 1 & 6 & \end{array} \right| \begin{array}{l} \downarrow \times 3 \\ = \end{array} \boxed{\text{III}} \left| \begin{array}{cccc|c} 1 & 2 & 3 & -1 & \\ 0 & 8 & 16 & 8 & \\ 0 & 5 & 9 & 16 & \\ -2 & 0 & 1 & 6 & \end{array} \right| \begin{array}{l} \downarrow \times 2 \\ = \end{array} \boxed{\text{III}} \left| \begin{array}{cccc|c} 1 & 2 & 3 & -1 & \\ 0 & 8 & 16 & 8 & \\ 0 & 5 & 9 & 16 & \\ 0 & 4 & 7 & 4 & \end{array} \right| \stackrel{3.3}{=} \left| \begin{array}{ccc|c} 8 & 16 & 8 & \\ 5 & 9 & 16 & \\ 4 & 7 & 4 & \end{array} \right| \end{array}$$

$$\boxed{\text{II}} = 8 \cdot \left| \begin{array}{ccc|c} 1 & 2 & 1 & \\ 5 & 9 & 16 & \\ 4 & 7 & 4 & \end{array} \right| \downarrow \times (-5)$$

# 例

$$\begin{array}{c} \boxed{\text{I}} \\ \left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \updownarrow \\ = - \end{array} \left| \begin{array}{ccc} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \downarrow \times (-3) \\ = - \end{array} \boxed{\text{III}} \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \downarrow \times 1 \\ = - \end{array} \boxed{\text{III}} \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right| \end{array}$$

$$\stackrel{3.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

# 例

$$\begin{array}{c} \boxed{\text{III}} \\ \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \begin{array}{l} \downarrow \times 3 \\ = \end{array} \boxed{\text{III}} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \begin{array}{l} \downarrow \times 2 \\ = \end{array} \boxed{\text{III}} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \stackrel{3.3}{=} \left| \begin{array}{ccc} 8 & 16 & 8 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right| \end{array}$$

$$\boxed{\text{II}} \left| \begin{array}{ccc} 1 & 2 & 1 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right| \begin{array}{l} \downarrow \times (-5) \\ = \end{array} \boxed{\text{III}} =$$

# 例

$$\left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \stackrel{\text{I}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\times(-3)} \stackrel{\text{III}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\times 1} \stackrel{\text{III}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right|$$

$$\stackrel{3.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

# 例

$$\left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\times 3} \stackrel{\text{III}}{=} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\times 2} \stackrel{\text{III}}{=} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \stackrel{3.3}{=} \left| \begin{array}{ccc} 8 & 16 & 8 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right|$$

$$\stackrel{\text{II}}{=} 8 \cdot \left| \begin{array}{ccc} 1 & 2 & 1 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right| \xrightarrow{\times(-5)} \stackrel{\text{III}}{=} 8 \cdot \left| \begin{array}{ccc} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 4 & 7 & 4 \end{array} \right|$$

# 例

$$\begin{vmatrix} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\text{I}} = - \begin{vmatrix} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\times(-3)} \begin{vmatrix} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\times 1} \begin{vmatrix} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{vmatrix}$$

$$\stackrel{3.3}{=} - \begin{vmatrix} 8 & 4 \\ -1 & 0 \end{vmatrix} = (-1) \cdot 4 = -4.$$

# 例

$$\begin{vmatrix} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{vmatrix} \xrightarrow{\times 3} \begin{vmatrix} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{vmatrix} \xrightarrow{\times 2} \begin{vmatrix} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{vmatrix} \stackrel{3.3}{=} \begin{vmatrix} 8 & 16 & 8 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 2 & 1 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{vmatrix} \xrightarrow{\times(-5)} \begin{vmatrix} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 4 & 7 & 4 \end{vmatrix} \xrightarrow{\times(-4)}$$

# 例

$$\begin{vmatrix} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\text{I}} = - \begin{vmatrix} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\times(-3)} \begin{vmatrix} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\times 1} \begin{vmatrix} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{vmatrix}$$

$$\stackrel{3.3}{=} - \begin{vmatrix} 8 & 4 \\ -1 & 0 \end{vmatrix} = (-1) \cdot 4 = -4.$$

# 例

$$\begin{vmatrix} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{vmatrix} \xrightarrow{\times 3} \begin{vmatrix} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{vmatrix} \xrightarrow{\times 2} \begin{vmatrix} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{vmatrix} \stackrel{3.3}{=} \begin{vmatrix} 8 & 16 & 8 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 2 & 1 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{vmatrix} \xrightarrow{\times(-5)} \begin{vmatrix} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 4 & 7 & 4 \end{vmatrix} \xrightarrow{\times(-4)} \begin{vmatrix} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 0 & 3 & -12 \end{vmatrix}$$

# 例

$$\begin{array}{c} \begin{array}{ccc|c} 3 & 2 & 1 & \\ 1 & -2 & -1 & \\ -1 & 1 & 1 & \end{array} \begin{array}{l} \updownarrow \\ = \\ \downarrow \end{array} \boxed{\text{I}} \\ \begin{array}{ccc|c} 1 & -2 & -1 & \\ 3 & 2 & 1 & \\ -1 & 1 & 1 & \end{array} \begin{array}{l} \\ \times(-3) \\ \downarrow \end{array} \boxed{\text{III}} \\ \begin{array}{ccc|c} 1 & -2 & -1 & \\ 0 & 8 & 4 & \\ -1 & 1 & 1 & \end{array} \begin{array}{l} \\ \\ \downarrow \end{array} \times 1 \boxed{\text{III}} \\ \begin{array}{ccc|c} 1 & -2 & -1 & \\ 0 & 8 & 4 & \\ 0 & -1 & 0 & \end{array} \end{array}$$

$$\stackrel{3.3}{=} - \begin{vmatrix} 8 & 4 \\ -1 & 0 \end{vmatrix} = (-1) \cdot 4 = -4.$$

# 例

$$\begin{array}{c} \begin{array}{cccc|c} 1 & 2 & 3 & -1 & \\ -3 & 2 & 7 & 11 & \\ 0 & 5 & 9 & 16 & \\ -2 & 0 & 1 & 6 & \end{array} \begin{array}{l} \downarrow \times 3 \\ = \\ \downarrow \end{array} \boxed{\text{III}} \\ \begin{array}{cccc|c} 1 & 2 & 3 & -1 & \\ 0 & 8 & 16 & 8 & \\ 0 & 5 & 9 & 16 & \\ -2 & 0 & 1 & 6 & \end{array} \begin{array}{l} \\ \times 2 \\ \downarrow \end{array} \boxed{\text{III}} \\ \begin{array}{cccc|c} 1 & 2 & 3 & -1 & \\ 0 & 8 & 16 & 8 & \\ 0 & 5 & 9 & 16 & \\ 0 & 4 & 7 & 4 & \end{array} \begin{array}{l} \\ \\ \stackrel{3.3}{=} \end{array} \begin{array}{ccc|c} 8 & 16 & 8 & \\ 5 & 9 & 16 & \\ 4 & 7 & 4 & \end{array} \end{array}$$

$$\begin{array}{c} \boxed{\text{II}} \\ = 8 \cdot \begin{array}{ccc|c} 1 & 2 & 1 & \\ 5 & 9 & 16 & \\ 4 & 7 & 4 & \end{array} \begin{array}{l} \downarrow \times(-5) \\ \\ \downarrow \end{array} \boxed{\text{III}} \\ = 8 \cdot \begin{array}{ccc|c} 1 & 2 & 1 & \\ 0 & -1 & 11 & \\ 4 & 7 & 4 & \end{array} \begin{array}{l} \\ \times(-4) \\ \downarrow \end{array} \boxed{\text{III}} \\ = 8 \cdot \begin{array}{ccc|c} 1 & 2 & 1 & \\ 0 & -1 & 11 & \\ 0 & -1 & 0 & \end{array} \end{array}$$

# 例

$$\begin{array}{c} \boxed{\text{I}} \\ \left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \updownarrow \\ = - \\ \downarrow \end{array} \left| \begin{array}{ccc} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \\ \times(-3) \\ \downarrow \end{array} \boxed{\text{III}} \\ = - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \\ \\ \downarrow \end{array} \times 1 \boxed{\text{III}} \\ = - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right| \end{array}$$

$$\stackrel{3.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

# 例

$$\begin{array}{c} \boxed{\text{III}} \\ \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \begin{array}{l} \downarrow \times 3 \\ = \\ \downarrow \end{array} \boxed{\text{III}} \\ = \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \begin{array}{l} \\ \times 2 \\ \downarrow \end{array} \boxed{\text{III}} \\ = \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \stackrel{3.3}{=} \left| \begin{array}{ccc} 8 & 16 & 8 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right| \end{array}$$

$$\begin{array}{c} \boxed{\text{II}} \\ = 8 \cdot \left| \begin{array}{ccc} 1 & 2 & 1 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right| \begin{array}{l} \downarrow \times(-5) \\ \\ \downarrow \end{array} \boxed{\text{III}} \\ = 8 \cdot \left| \begin{array}{ccc} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 4 & 7 & 4 \end{array} \right| \begin{array}{l} \\ \times(-4) \\ \downarrow \end{array} \boxed{\text{III}} \\ = 8 \cdot \left| \begin{array}{ccc} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 0 & -1 & 0 \end{array} \right| \stackrel{3.3}{=} \end{array}$$

# 例

$$\begin{array}{c} \boxed{\text{I}} \\ \left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \updownarrow \\ = - \end{array} \left| \begin{array}{ccc} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \\ \downarrow \times (-3) \end{array} \boxed{\text{III}} \\ = - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \\ \downarrow \times 1 \end{array} \boxed{\text{III}} \\ = - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right| \end{array}$$

$$\stackrel{3.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

# 例

$$\begin{array}{c} \boxed{\text{III}} \\ \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \begin{array}{l} \\ \downarrow \times 3 \end{array} \boxed{\text{III}} \\ = \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \begin{array}{l} \\ \downarrow \times 2 \end{array} \boxed{\text{III}} \\ = \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \stackrel{3.3}{=} \left| \begin{array}{ccc} 8 & 16 & 8 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right| \end{array}$$

$$\begin{array}{c} \boxed{\text{II}} \\ = 8 \cdot \left| \begin{array}{ccc} 1 & 2 & 1 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right| \begin{array}{l} \\ \downarrow \times (-5) \end{array} \boxed{\text{III}} \\ = 8 \cdot \left| \begin{array}{ccc} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 4 & 7 & 4 \end{array} \right| \begin{array}{l} \\ \downarrow \times (-4) \end{array} \boxed{\text{III}} \\ = 8 \cdot \left| \begin{array}{ccc} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 0 & -1 & 0 \end{array} \right| \stackrel{3.3}{=} 8 \cdot \left| \begin{array}{cc} -1 & 11 \\ -1 & 0 \end{array} \right| \end{array}$$



# 例

$$\begin{vmatrix} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\text{I}} = - \begin{vmatrix} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\times(-3)} \begin{vmatrix} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\times 1} \begin{vmatrix} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{vmatrix}$$

$$\stackrel{3.3}{=} - \begin{vmatrix} 8 & 4 \\ -1 & 0 \end{vmatrix} = (-1) \cdot 4 = -4.$$

# 例

$$\begin{vmatrix} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{vmatrix} \xrightarrow{\times 3} \begin{vmatrix} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{vmatrix} \xrightarrow{\times 2} \begin{vmatrix} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{vmatrix} \stackrel{3.3}{=} \begin{vmatrix} 8 & 16 & 8 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{vmatrix}$$

$$\stackrel{\text{II}}{=} 8 \cdot \begin{vmatrix} 1 & 2 & 1 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{vmatrix} \xrightarrow{\times(-5)} \begin{vmatrix} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 4 & 7 & 4 \end{vmatrix} \xrightarrow{\times(-4)} \begin{vmatrix} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 0 & -1 & 0 \end{vmatrix} \stackrel{3.3}{=} 8 \cdot \begin{vmatrix} -1 & 11 \\ -1 & 0 \end{vmatrix}$$

$$= 8 \cdot 11$$

# 例

$$\begin{vmatrix} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\text{I}} = - \begin{vmatrix} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\times(-3)} \begin{vmatrix} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\times 1} \begin{vmatrix} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{vmatrix}$$

$$\stackrel{3.3}{=} - \begin{vmatrix} 8 & 4 \\ -1 & 0 \end{vmatrix} = (-1) \cdot 4 = -4.$$

# 例

$$\begin{vmatrix} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{vmatrix} \xrightarrow{\times 3} \begin{vmatrix} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{vmatrix} \xrightarrow{\times 2} \begin{vmatrix} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{vmatrix} \stackrel{3.3}{=} \begin{vmatrix} 8 & 16 & 8 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{vmatrix}$$

$$\stackrel{\text{II}}{=} 8 \cdot \begin{vmatrix} 1 & 2 & 1 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{vmatrix} \xrightarrow{\times(-5)} \begin{vmatrix} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 4 & 7 & 4 \end{vmatrix} \xrightarrow{\times(-4)} \begin{vmatrix} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 0 & -1 & 0 \end{vmatrix} \stackrel{3.3}{=} 8 \cdot \begin{vmatrix} -1 & 11 \\ -1 & 0 \end{vmatrix}$$

$$= 8 \cdot 11 = 88.$$

# 例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

# 例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

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$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix}$$

# 例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix} = \boxed{\text{III}}$$

# 例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix} \begin{matrix} \uparrow \\ \uparrow \end{matrix} \begin{matrix} \text{III} \\ \text{III} \end{matrix} = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

# 例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix} = \boxed{\text{III}} \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$\boxed{\text{II}}$

=



# 例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix} = \boxed{\text{III}} \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$$\boxed{\text{II}} = (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ y & x & y \\ z & z & x \end{vmatrix}$$

# 例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix} = \begin{matrix} \text{III} \\ \\ \end{matrix} \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$$\begin{matrix} \text{II} \\ \\ \end{matrix} = (x+y+z) \begin{vmatrix} \mathbf{1} & 1 & 1 \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \downarrow \times (-y) \\ \downarrow \times (-z) \end{matrix}$$

# 例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix} \begin{matrix} \text{III} \\ \\ \end{matrix} = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$$\begin{matrix} \text{II} \\ \\ \end{matrix} = (x+y+z) \begin{vmatrix} \mathbf{1} & 1 & 1 \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \downarrow \times (-y) \\ \downarrow \times (-z) \end{matrix} \begin{matrix} \text{III} \\ \\ \end{matrix} =$$

# 例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix} \begin{matrix} \text{III} \\ \\ \end{matrix} = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$$\begin{matrix} \text{II} \\ \\ \end{matrix} = (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \downarrow \times (-y) \\ \downarrow \times (-z) \end{matrix} \begin{matrix} \text{III} \\ \\ \end{matrix} = (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ 0 & x-y & 0 \\ 0 & 0 & x-z \end{vmatrix}$$

# 例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix} \overset{\text{III}}{=} \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$$\overset{\text{II}}{=} (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \downarrow \times (-y) \\ \downarrow \times (-z) \end{matrix} \overset{\text{III}}{=} (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ 0 & x-y & 0 \\ 0 & 0 & x-z \end{vmatrix}$$

3.3

# 例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \xrightarrow{\times 1} \xrightarrow{\times 1} \boxed{\text{III}} = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$$\boxed{\text{II}} = (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ y & x & y \\ z & z & x \end{vmatrix} \xrightarrow{\times(-y)} \xrightarrow{\times(-z)} \boxed{\text{III}} = (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ 0 & x-y & 0 \\ 0 & 0 & x-z \end{vmatrix}$$

$$\stackrel{3.3}{=} (x+y+z) \begin{vmatrix} x-y & 0 \\ 0 & x-z \end{vmatrix}$$

# 例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix} \begin{matrix} \text{III} \\ \\ \end{matrix} = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$$\begin{matrix} \text{II} \\ \\ \end{matrix} = (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \downarrow \times (-y) \\ \downarrow \times (-z) \end{matrix} \begin{matrix} \text{III} \\ \\ \end{matrix} = (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ 0 & x-y & 0 \\ 0 & 0 & x-z \end{vmatrix}$$

$$\stackrel{3.3}{=} (x+y+z) \begin{vmatrix} x-y & 0 \\ 0 & x-z \end{vmatrix} = (x+y+z)(x-y)(x-z).$$

# 例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix} \begin{matrix} \text{III} \\ \\ \end{matrix} = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$$\begin{matrix} \text{II} \\ \\ \end{matrix} = (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \downarrow \times (-y) \\ \downarrow \times (-z) \end{matrix} \begin{matrix} \text{III} \\ \\ \end{matrix} = (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ 0 & x-y & 0 \\ 0 & 0 & x-z \end{vmatrix}$$

$$\stackrel{3.3}{=} (x+y+z) \begin{vmatrix} x-y & 0 \\ 0 & x-z \end{vmatrix} = (x+y+z)(x-y)(x-z).$$

# 注意

$n$  次正方行列  $A$  に対して、定理 3.6 を  $n$  回使うと、  
 $|kA| = k^n |A|$ .  $\cdots |kA| \neq k|A|$  ( $n \geq 2$ ) に注意する