

はじめに (数学基礎 B2)

数学基礎 B = 線形代数

教科書 「要点明解 線形数学」 培風館

(第1章 ベクトル)

(第2章 行列)

(第3章 連立1次方程式)

▶ 第4章 行列式

▶ 第5章 行列の対角化

講義の情報

<http://mathweb.sc.niigata-u.ac.jp/~hoshi/teaching-j.html>

シラバス [LINK](#)

▶ ノートを取りながら講義を聴くこと。

(ノートを回収して確認する可能性があります)

▶ 講義 → 小テスト (理解度確認テスト, 学務情報システム内)

定理 4.6

n 次正方行列 A のある行を k 倍した行列 A'' に対して, $|A''| = k|A|$.

定理 4.6

n 次正方行列 A のある行を k 倍した行列 A'' に対して, $|A''| = k|A|$.

例

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, A'' = \begin{pmatrix} ka & kb \\ c & d \end{pmatrix}$$

定理 4.6

n 次正方行列 A のある行を k 倍した行列 A'' に対して, $|A''| = k|A|$.

例

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, A'' = \begin{pmatrix} ka & kb \\ c & d \end{pmatrix} \Rightarrow |A''| = kad - kbc = k(ad - bc) = k|A|.$$

定理 4.7

n 次正方行列 A のある行が行ベクトルの和 $\mathbf{a} + \mathbf{a}'$

$$\Rightarrow |A| = \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \mathbf{a} + \mathbf{a}' \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \mathbf{a} \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} + \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \mathbf{a}' \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} .$$

定理 4.7

n 次正方行列 A のある行が行ベクトルの和 $\mathbf{a} + \mathbf{a}'$

$$\Rightarrow |A| = \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \mathbf{a} + \mathbf{a}' \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \mathbf{a} \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} + \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \mathbf{a}' \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} .$$

例

$$A = \begin{pmatrix} \mathbf{a} + \mathbf{a}' & \mathbf{b} + \mathbf{b}' \\ c & d \end{pmatrix} \Rightarrow$$

定理 4.7

n 次正方行列 A のある行が行ベクトルの和 $\mathbf{a} + \mathbf{a}'$

$$\Rightarrow |A| = \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \mathbf{a} + \mathbf{a}' \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \mathbf{a} \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} + \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \mathbf{a}' \\ a_{n1} & \cdots & a_{nn} \end{vmatrix}.$$

例

$$A = \begin{pmatrix} a + a' & b + b' \\ c & d \end{pmatrix} \Rightarrow |A| = (a + a')d - (b + b')c$$

定理 4.7

n 次正方行列 A のある行が行ベクトルの和 $\mathbf{a} + \mathbf{a}'$

$$\Rightarrow |A| = \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \mathbf{a} + \mathbf{a}' \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \mathbf{a} \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} + \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \mathbf{a}' \\ a_{n1} & \cdots & a_{nn} \end{vmatrix}.$$

例

$$A = \begin{pmatrix} \mathbf{a} + \mathbf{a}' & \mathbf{b} + \mathbf{b}' \\ c & d \end{pmatrix} \Rightarrow |A| = (a + a')d - (b + b')c$$
$$= (ad - bc) + (a'd - b'c) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} a' & b' \\ c & d \end{vmatrix}.$$

定理 4.8

n 次正方行列 A のある (i) 行に他の行 (j) 行の k 倍を加えた行列 A''' に対して, $|A'''| = |A|$.

定理 4.8

n 次正方行列 A のある (i) 行に他の行 (j) 行の k 倍を加えた行列 A''' に対して, $|A'''| = |A|$.

$$\therefore |A'''| = \begin{vmatrix} a_1 \\ a_i + k a_j \\ a_j \\ a_n \end{vmatrix}$$

定理 4.8

n 次正方行列 A のある (i) 行に他の行 (j) 行の k 倍を加えた行列 A''' に対して, $|A'''| = |A|$.

$$\therefore |A'''| = \begin{vmatrix} a_1 & & & \\ a_i + k a_j & & & \\ a_j & & & \\ a_n & & & \end{vmatrix} \stackrel{4.7}{=} \begin{vmatrix} a_1 & & & \\ a_i & & & \\ a_j & & & \\ a_n & & & \end{vmatrix} + \begin{vmatrix} a_1 & & & \\ k a_j & & & \\ a_j & & & \\ a_n & & & \end{vmatrix}$$

定理 4.8

n 次正方行列 A のある (i) 行に他の行 (j) 行の k 倍を加えた行列 A''' に対して, $|A'''| = |A|$.

$$\begin{aligned} \therefore |A'''| &= \begin{vmatrix} a_1 & & & & \\ a_i + k a_j & & & & \\ a_j & & & & \\ & & & & \\ a_n & & & & \end{vmatrix} \stackrel{4.7}{=} \begin{vmatrix} a_1 & & & & \\ a_i & & & & \\ a_j & & & & \\ & & & & \\ a_n & & & & \end{vmatrix} + \begin{vmatrix} a_1 & & & & \\ k a_j & & & & \\ a_j & & & & \\ & & & & \\ a_n & & & & \end{vmatrix} \stackrel{4.6}{=} |A| + k \begin{vmatrix} a_1 & & & & \\ a_j & & & & \\ a_j & & & & \\ & & & & \\ a_n & & & & \end{vmatrix} \end{aligned}$$

定理 4.8

n 次正方行列 A のある (i) 行に他の行 (j) 行の k 倍を加えた行列 A''' に対して, $|A'''| = |A|$.

$$\begin{aligned} \therefore |A'''| &= \begin{vmatrix} a_1 & & & \\ a_i + k a_j & & & \\ & & & \\ a_j & & & \\ & & & \\ a_n & & & \end{vmatrix} \stackrel{4.7}{=} \begin{vmatrix} a_1 & & & \\ a_i & & & \\ & & & \\ a_j & & & \\ & & & \\ a_n & & & \end{vmatrix} + \begin{vmatrix} a_1 & & & \\ k a_j & & & \\ & & & \\ a_j & & & \\ & & & \\ a_n & & & \end{vmatrix} \stackrel{4.6}{=} |A| + k \begin{vmatrix} a_1 & & & \\ a_j & & & \\ & & & \\ a_j & & & \\ & & & \\ a_n & & & \end{vmatrix} \end{aligned}$$

$$\stackrel{4.5}{=} |A| + k \cdot 0 = |A|.$$

まとめると …

まとめると … 3つの行基本変形 I, II, III に対して,

まとめると … 3つの行基本変形 I, II, III に対して,

I 行列 A の2つの行を入れかえた行列を A'

まとめると … 3つの行基本変形 I, II, III に対して,

I 行列 A の2つの行を入れかえた行列を A'

II 行列 A のある行を k 倍した行列を A''

まとめると … 3つの行基本変形 I, II, III に対して,

I 行列 A の2つの行を入れかえた行列を A'

II 行列 A のある行を k 倍した行列を A''

III 行列 A のある行に他の行の k 倍をたした行列を A'''

まとめると … 3つの行基本変形 I, II, III に対して,

I 行列 A の2つの行を入れかえた行列を A'

II 行列 A のある行を k 倍した行列を A''

III 行列 A のある行に他の行の k 倍をたした行列を A'''

とすると,

まとめると … 3つの行基本変形 I, II, III に対して,

I 行列 A の2つの行を入れかえた行列を A'

II 行列 A のある行を k 倍した行列を A''

III 行列 A のある行に他の行の k 倍をたした行列を A'''

とすると,

$$|A'| = -|A| \quad \cdots (1)$$

$$|A''| = k|A| \quad \cdots (2)$$

$$|A'''| = |A| \quad \cdots (3)$$

まとめると … 3つの行基本変形 I, II, III に対して,

I 行列 A の2つの行を入れかえた行列を A'

II 行列 A のある行を k 倍した行列を A''

III 行列 A のある行に他の行の k 倍をたした行列を A'''

とすると,

$$|A'| = -|A| \quad \cdots (1)$$

$$|A''| = k|A| \quad \cdots (2)$$

$$|A'''| = |A| \quad \cdots (3)$$

となる.

まとめると … 3つの行基本変形 I, II, III に対して,

I 行列 A の2つの行を入れかえた行列を A'

II 行列 A のある行を k 倍した行列を A''

III 行列 A のある行に他の行の k 倍をたした行列を A'''

とすると,

$$|A'| = -|A| \quad \cdots (1)$$

$$|A''| = k|A| \quad \cdots (2)$$

$$|A'''| = |A| \quad \cdots (3)$$

となる. この (1), (2), (3) を用いれば, 行基本変形 I, II, III によって,

まとめると … 3つの行基本変形 I, II, III に対して,

I 行列 A の2つの行を入れかえた行列を A'

II 行列 A のある行を k 倍した行列を A''

III 行列 A のある行に他の行の k 倍をたした行列を A'''

とすると,

$$|A'| = -|A| \quad \cdots (1)$$

$$|A''| = k|A| \quad \cdots (2)$$

$$|A'''| = |A| \quad \cdots (3)$$

となる. この (1), (2), (3) を用いれば, 行基本変形 I, II, III によって,

$$A \rightarrow \cdots \rightarrow \begin{pmatrix} a & * & \cdots & * \\ 0 & \color{red}{\square} & & \\ \vdots & & & \\ 0 & \color{red}{\square} & & \end{pmatrix} \text{の形にできれば, (定理 4.3 より)}$$

まとめると … 3つの行基本変形 I, II, III に対して,

I 行列 A の2つの行を入れかえた行列を A'

II 行列 A のある行を k 倍した行列を A''

III 行列 A のある行に他の行の k 倍をたした行列を A'''

とすると,

$$|A'| = -|A| \quad \cdots (1)$$

$$|A''| = k|A| \quad \cdots (2)$$

$$|A'''| = |A| \quad \cdots (3)$$

となる. この (1), (2), (3) を用いれば, 行基本変形 I, II, III によって,

$$A \rightarrow \cdots \rightarrow \begin{pmatrix} a & * & \cdots & * \\ 0 & \color{red}{\boxed{B}} \\ \vdots & & & \\ 0 & & & \end{pmatrix} \text{の形にできれば, (定理 4.3 より)}$$

$|A|$ を $a \cdot |B|$ を用いて計算できる.

例

$$\left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right|$$

例

$$\left| \begin{array}{ccc|c} 3 & 2 & 1 & \updownarrow \\ 1 & -2 & -1 & \\ -1 & 1 & 1 & \end{array} \right|$$

例

$$\left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{c} \updownarrow \\ \square \\ = \end{array} \text{I}$$

例

$$\left| \begin{array}{ccc|c} 3 & 2 & 1 & \boxed{I} \\ 1 & -2 & -1 & \\ -1 & 1 & 1 & \end{array} \right| \begin{array}{c} \updownarrow \\ = - \end{array} \left| \begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 3 & 2 & 1 & \\ -1 & 1 & 1 & \end{array} \right|$$

例

$$\left| \begin{array}{ccc|c} 3 & 2 & 1 & \boxed{I} \\ 1 & -2 & -1 & \\ -1 & 1 & 1 & \end{array} \right| \begin{array}{l} \updownarrow \\ = - \end{array} \left| \begin{array}{ccc|c} 1 & -2 & -1 & \\ 3 & 2 & 1 & \\ -1 & 1 & 1 & \end{array} \right| \begin{array}{l} \\ \downarrow \times (-3) \end{array}$$

例

$$\left| \begin{array}{ccc|c} 3 & 2 & 1 & \boxed{\text{I}} \\ 1 & -2 & -1 & = \\ -1 & 1 & 1 & \end{array} \right| \begin{array}{l} \updownarrow \\ = - \end{array} \left| \begin{array}{ccc|c} \mathbf{1} & -2 & -1 & \boxed{\text{III}} \\ 3 & 2 & 1 & = \\ -1 & 1 & 1 & \end{array} \right| \begin{array}{l} \downarrow \times (-3) \\ \end{array}$$

例

$$\left| \begin{array}{ccc|c} 3 & 2 & 1 & \boxed{\text{I}} \\ 1 & -2 & -1 & \\ -1 & 1 & 1 & \end{array} \right| \begin{array}{l} \updownarrow \\ = - \end{array} \left| \begin{array}{ccc|c} 1 & -2 & -1 & \\ 3 & 2 & 1 & \\ -1 & 1 & 1 & \end{array} \right| \begin{array}{l} \downarrow \times (-3) \\ = - \end{array} \left| \begin{array}{ccc|c} 1 & -2 & -1 & \boxed{\text{III}} \\ 0 & 8 & 4 & \\ -1 & 1 & 1 & \end{array} \right|$$

例

$$\left| \begin{array}{ccc|c} 3 & 2 & 1 & \boxed{\text{I}} \\ 1 & -2 & -1 & \\ -1 & 1 & 1 & \end{array} \right| \begin{array}{l} \updownarrow \\ = - \end{array} \left| \begin{array}{ccc|c} 1 & -2 & -1 & \\ 3 & 2 & 1 & \\ -1 & 1 & 1 & \end{array} \right| \begin{array}{l} \downarrow \times (-3) \\ = - \end{array} \boxed{\text{III}} \left| \begin{array}{ccc|c} 1 & -2 & -1 & \\ 0 & 8 & 4 & \\ -1 & 1 & 1 & \end{array} \right| \begin{array}{l} \downarrow \times 1 \\ \downarrow \end{array}$$

例

$$\left| \begin{array}{ccc|c} 3 & 2 & 1 & \boxed{\text{I}} \\ 1 & -2 & -1 & \\ -1 & 1 & 1 & \end{array} \right| \begin{array}{l} \updownarrow \\ = - \end{array} \left| \begin{array}{ccc|c} 1 & -2 & -1 & \\ 3 & 2 & 1 & \\ -1 & 1 & 1 & \end{array} \right| \begin{array}{l} \downarrow \times (-3) \\ = - \end{array} \left| \begin{array}{ccc|c} 1 & -2 & -1 & \boxed{\text{III}} \\ 0 & 8 & 4 & \\ -1 & 1 & 1 & \end{array} \right| \begin{array}{l} \downarrow \times 1 \\ = \end{array} \left| \begin{array}{ccc|c} 1 & -2 & -1 & \boxed{\text{III}} \\ 0 & 8 & 4 & \\ -1 & 1 & 1 & \end{array} \right|$$

例

$$\begin{array}{ccc|c} 3 & 2 & 1 & \boxed{\text{I}} \\ 1 & -2 & -1 & \\ -1 & 1 & 1 & \end{array} \begin{array}{c} \updownarrow \\ = - \\ \end{array} \begin{array}{ccc|c} 1 & -2 & -1 & \\ 3 & 2 & 1 & \\ -1 & 1 & 1 & \end{array} \begin{array}{c} \downarrow \times (-3) \\ \\ \end{array} \begin{array}{ccc|c} \boxed{\text{III}} & & & \\ = - & & & \\ \begin{array}{ccc|c} 1 & -2 & -1 & \\ 0 & 8 & 4 & \\ -1 & 1 & 1 & \end{array} & & & \end{array} \begin{array}{c} \downarrow \times 1 \\ \\ \end{array} \begin{array}{ccc|c} \boxed{\text{III}} & & & \\ = - & & & \\ \begin{array}{ccc|c} 1 & -2 & -1 & \\ 0 & 8 & 4 & \\ 0 & -1 & 0 & \end{array} & & & \end{array}$$

例

$$\left| \begin{array}{ccc|c} 3 & 2 & 1 & \boxed{\text{I}} \\ 1 & -2 & -1 & \\ -1 & 1 & 1 & \end{array} \right| \begin{array}{l} \updownarrow \\ = - \end{array} \left| \begin{array}{ccc|c} \mathbf{1} & -2 & -1 & \\ 3 & 2 & 1 & \\ -1 & 1 & 1 & \end{array} \right| \begin{array}{l} \downarrow \times (-3) \\ = - \end{array} \boxed{\text{III}} \left| \begin{array}{ccc|c} \mathbf{1} & -2 & -1 & \\ 0 & 8 & 4 & \\ -1 & 1 & 1 & \end{array} \right| \begin{array}{l} \downarrow \times 1 \\ = - \end{array} \boxed{\text{III}} \left| \begin{array}{ccc|c} \mathbf{1} & -2 & -1 & \\ 0 & 8 & 4 & \\ 0 & -1 & 0 & \end{array} \right|$$

4.3

≡

例

$$\begin{array}{c} \left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \updownarrow \\ \text{I} \end{array} \\ = - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \downarrow \times (-3) \\ \text{III} \end{array} \\ = - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \downarrow \times 1 \\ \text{III} \end{array} \\ = - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right| \end{array}$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right|$$

例

$$\begin{array}{c} \left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \updownarrow \\ \text{I} \end{array} \\ \text{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \downarrow \times (-3) \\ \text{III} \end{array} \\ \text{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \downarrow \times 1 \\ \text{III} \end{array} \\ \text{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right| \end{array}$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\left| \begin{array}{ccc|c} 3 & 2 & 1 & \\ 1 & -2 & -1 & \\ -1 & 1 & 1 & \end{array} \right| \stackrel{\text{I}}{\rightleftharpoons} = - \left| \begin{array}{ccc|c} 1 & -2 & -1 & \\ 3 & 2 & 1 & \\ -1 & 1 & 1 & \end{array} \right| \stackrel{\text{III}}{\downarrow \times (-3)} = - \left| \begin{array}{ccc|c} 1 & -2 & -1 & \\ 0 & 8 & 4 & \\ -1 & 1 & 1 & \end{array} \right| \stackrel{\text{III}}{\downarrow \times 1} = - \left| \begin{array}{ccc|c} 1 & -2 & -1 & \\ 0 & 8 & 4 & \\ 0 & -1 & 0 & \end{array} \right|$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc|c} 8 & 4 & \\ -1 & 0 & \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\left| \begin{array}{cccc|c} 1 & 2 & 3 & -1 & \\ -3 & 2 & 7 & 11 & \\ 0 & 5 & 9 & 16 & \\ -2 & 0 & 1 & 6 & \end{array} \right|$$

例

$$\begin{array}{c} \left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \updownarrow \\ \text{I} \end{array} \\ = - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \downarrow \times (-3) \\ \text{III} \end{array} \\ = - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \downarrow \times 1 \\ \text{III} \end{array} \\ = - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right| \end{array}$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \begin{array}{l} \downarrow \times 3 \end{array}$$

例

$$\begin{vmatrix} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\text{I}} = - \begin{vmatrix} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\times(-3)} \text{III} = - \begin{vmatrix} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\times 1} \text{III} = - \begin{vmatrix} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{vmatrix}$$

$$\stackrel{4.3}{=} - \begin{vmatrix} 8 & 4 \\ -1 & 0 \end{vmatrix} = (-1) \cdot 4 = -4.$$

例

$$\begin{vmatrix} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{vmatrix} \xrightarrow{\times 3} \text{III} =$$

例

$$\left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \stackrel{\boxed{\text{I}}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \downarrow \times (-3) \stackrel{\boxed{\text{III}}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \downarrow \times 1 \stackrel{\boxed{\text{III}}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right|$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \downarrow \times 3 \stackrel{\boxed{\text{III}}}{=} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right|$$

例

$$\begin{vmatrix} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\text{I}} = - \begin{vmatrix} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\times(-3)} \begin{vmatrix} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\times 1} \begin{vmatrix} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{vmatrix}$$

$$\stackrel{4.3}{=} - \begin{vmatrix} 8 & 4 \\ -1 & 0 \end{vmatrix} = (-1) \cdot 4 = -4.$$

例

$$\begin{vmatrix} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{vmatrix} \xrightarrow{\times 3} \begin{vmatrix} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{vmatrix} \xrightarrow{\times 2}$$

例

$$\left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \stackrel{\text{I}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \times (-3) \stackrel{\text{III}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \times 1 \stackrel{\text{III}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right|$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \stackrel{\text{III}}{=} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \stackrel{\text{III}}{=} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \times 2$$

例

$$\begin{array}{ccc|c} 3 & 2 & 1 & \\ 1 & -2 & -1 & \\ -1 & 1 & 1 & \end{array} \begin{array}{l} \updownarrow \\ = - \\ \end{array} \boxed{\text{I}} \begin{array}{ccc|c} 1 & -2 & -1 & \\ 3 & 2 & 1 & \\ -1 & 1 & 1 & \end{array} \begin{array}{l} \downarrow \times (-3) \\ = - \\ \end{array} \boxed{\text{III}} \begin{array}{ccc|c} 1 & -2 & -1 & \\ 0 & 8 & 4 & \\ -1 & 1 & 1 & \end{array} \begin{array}{l} \downarrow \times 1 \\ = - \\ \end{array} \boxed{\text{III}} \begin{array}{ccc|c} 1 & -2 & -1 & \\ 0 & 8 & 4 & \\ 0 & -1 & 0 & \end{array}$$

$$\stackrel{4.3}{=} - \begin{vmatrix} 8 & 4 \\ -1 & 0 \end{vmatrix} = (-1) \cdot 4 = -4.$$

例

$$\begin{array}{cccc|c} 1 & 2 & 3 & -1 & \\ -3 & 2 & 7 & 11 & \\ 0 & 5 & 9 & 16 & \\ -2 & 0 & 1 & 6 & \end{array} \begin{array}{l} \downarrow \times 3 \\ = \\ \end{array} \boxed{\text{III}} \begin{array}{cccc|c} 1 & 2 & 3 & -1 & \\ 0 & 8 & 16 & 8 & \\ 0 & 5 & 9 & 16 & \\ -2 & 0 & 1 & 6 & \end{array} \begin{array}{l} \downarrow \times 2 \\ = \\ \end{array} \boxed{\text{III}} \begin{array}{cccc|c} 1 & 2 & 3 & -1 & \\ 0 & 8 & 16 & 8 & \\ 0 & 5 & 9 & 16 & \\ 0 & 4 & 7 & 4 & \end{array}$$

例

$$\left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \stackrel{\text{I}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\times(-3)} \stackrel{\text{III}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\times 1} \stackrel{\text{III}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right|$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\times 3} \stackrel{\text{III}}{=} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\times 2} \stackrel{\text{III}}{=} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \stackrel{4.3}{=}$$

例

$$\left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \stackrel{\text{I}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\times(-3)} \stackrel{\text{III}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\times 1} \stackrel{\text{III}}{=} - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right|$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\times 3} \stackrel{\text{III}}{=} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\times 2} \stackrel{\text{III}}{=} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \stackrel{4.3}{=} \left| \begin{array}{ccc} 8 & 16 & 8 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right|$$

例

$$\begin{array}{c} \left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \boxed{\text{I}} \\ \updownarrow \\ = - \end{array} \left| \begin{array}{ccc} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \times (-3) \\ \downarrow \\ \boxed{\text{III}} \\ = - \end{array} \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \times 1 \\ \downarrow \\ \boxed{\text{III}} \\ = - \end{array} \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right| \end{array}$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\begin{array}{c} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \begin{array}{l} \times 3 \\ \downarrow \\ \boxed{\text{III}} \\ = \end{array} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \begin{array}{l} \times 2 \\ \downarrow \\ \boxed{\text{III}} \\ = \end{array} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \stackrel{4.3}{=} \left| \begin{array}{ccc} 8 & 16 & 8 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right| \end{array}$$

$$\boxed{\text{II}} \\ =$$

例

$$\begin{array}{c} \boxed{\text{I}} \\ \left| \begin{array}{ccc|c} 3 & 2 & 1 & \\ 1 & -2 & -1 & \\ -1 & 1 & 1 & \end{array} \right| \begin{array}{l} \updownarrow \\ = - \end{array} \left| \begin{array}{ccc|c} 1 & -2 & -1 & \\ 3 & 2 & 1 & \\ -1 & 1 & 1 & \end{array} \right| \begin{array}{l} \downarrow \times (-3) \\ = - \end{array} \boxed{\text{III}} \left| \begin{array}{ccc|c} 1 & -2 & -1 & \\ 0 & 8 & 4 & \\ -1 & 1 & 1 & \end{array} \right| \begin{array}{l} \downarrow \times 1 \\ = - \end{array} \boxed{\text{III}} \left| \begin{array}{ccc|c} 1 & -2 & -1 & \\ 0 & 8 & 4 & \\ 0 & -1 & 0 & \end{array} \right| \end{array}$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc|c} 8 & 4 & \\ -1 & 0 & \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\begin{array}{c} \boxed{\text{III}} \\ \left| \begin{array}{cccc|c} 1 & 2 & 3 & -1 & \\ -3 & 2 & 7 & 11 & \\ 0 & 5 & 9 & 16 & \\ -2 & 0 & 1 & 6 & \end{array} \right| \begin{array}{l} \downarrow \times 3 \\ = \end{array} \boxed{\text{III}} \left| \begin{array}{cccc|c} 1 & 2 & 3 & -1 & \\ 0 & 8 & 16 & 8 & \\ 0 & 5 & 9 & 16 & \\ -2 & 0 & 1 & 6 & \end{array} \right| \begin{array}{l} \downarrow \times 2 \\ = \end{array} \boxed{\text{III}} \left| \begin{array}{cccc|c} 1 & 2 & 3 & -1 & \\ 0 & 8 & 16 & 8 & \\ 0 & 5 & 9 & 16 & \\ 0 & 4 & 7 & 4 & \end{array} \right| \stackrel{4.3}{=} \left| \begin{array}{ccc|c} 8 & 16 & 8 & \\ 5 & 9 & 16 & \\ 4 & 7 & 4 & \end{array} \right| \end{array}$$

$$\boxed{\text{II}} = 8 \cdot \left| \begin{array}{ccc|c} 1 & 2 & 1 & \\ 5 & 9 & 16 & \\ 4 & 7 & 4 & \end{array} \right|$$

例

$$\left| \begin{array}{ccc|c} 3 & 2 & 1 & \\ 1 & -2 & -1 & \\ -1 & 1 & 1 & \end{array} \right| \stackrel{\text{I}}{=} - \left| \begin{array}{ccc|c} 1 & -2 & -1 & \\ 3 & 2 & 1 & \\ -1 & 1 & 1 & \end{array} \right| \xrightarrow{\times(-3)} \stackrel{\text{III}}{=} - \left| \begin{array}{ccc|c} 1 & -2 & -1 & \\ 0 & 8 & 4 & \\ -1 & 1 & 1 & \end{array} \right| \xrightarrow{\times 1} \stackrel{\text{III}}{=} - \left| \begin{array}{ccc|c} 1 & -2 & -1 & \\ 0 & 8 & 4 & \\ 0 & -1 & 0 & \end{array} \right|$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc|c} 8 & 4 & \\ -1 & 0 & \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\left| \begin{array}{cccc|c} 1 & 2 & 3 & -1 & \\ -3 & 2 & 7 & 11 & \\ 0 & 5 & 9 & 16 & \\ -2 & 0 & 1 & 6 & \end{array} \right| \xrightarrow{\times 3} \stackrel{\text{III}}{=} \left| \begin{array}{cccc|c} 1 & 2 & 3 & -1 & \\ 0 & 8 & 16 & 8 & \\ 0 & 5 & 9 & 16 & \\ -2 & 0 & 1 & 6 & \end{array} \right| \xrightarrow{\times 2} \stackrel{\text{III}}{=} \left| \begin{array}{cccc|c} 1 & 2 & 3 & -1 & \\ 0 & 8 & 16 & 8 & \\ 0 & 5 & 9 & 16 & \\ 0 & 4 & 7 & 4 & \end{array} \right| \stackrel{4.3}{=} \left| \begin{array}{ccc|c} 8 & 16 & 8 & \\ 5 & 9 & 16 & \\ 4 & 7 & 4 & \end{array} \right|$$

$$\stackrel{\text{II}}{=} 8 \cdot \left| \begin{array}{ccc|c} 1 & 2 & 1 & \\ 5 & 9 & 16 & \\ 4 & 7 & 4 & \end{array} \right| \xrightarrow{\times(-5)}$$

例

$$\begin{array}{c} \boxed{\text{I}} \\ \left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \updownarrow \\ = - \end{array} \left| \begin{array}{ccc} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \downarrow \times (-3) \\ = - \end{array} \boxed{\text{III}} \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \downarrow \times 1 \\ = - \end{array} \boxed{\text{III}} \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right| \end{array}$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\begin{array}{c} \boxed{\text{III}} \\ \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \begin{array}{l} \downarrow \times 3 \\ = \end{array} \boxed{\text{III}} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \begin{array}{l} \downarrow \times 2 \\ = \end{array} \boxed{\text{III}} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \stackrel{4.3}{=} \left| \begin{array}{ccc} 8 & 16 & 8 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right| \end{array}$$

$$\boxed{\text{II}} \left| \begin{array}{ccc} 1 & 2 & 1 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right| \begin{array}{l} \downarrow \times (-5) \\ = \end{array} \boxed{\text{III}} \left| \begin{array}{ccc} 1 & 2 & 1 \\ 0 & 1 & 11 \\ 0 & 3 & 16 \end{array} \right|$$

例

$$\begin{array}{c} \left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \boxed{\text{I}} \\ \updownarrow \\ = - \end{array} \left| \begin{array}{ccc} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \times(-3) \\ \downarrow \\ \boxed{\text{III}} \\ = - \end{array} \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \times 1 \\ \downarrow \\ \boxed{\text{III}} \\ = - \end{array} \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right| \end{array}$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\begin{array}{c} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \begin{array}{l} \times 3 \\ \downarrow \\ \boxed{\text{III}} \\ = \end{array} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \begin{array}{l} \times 2 \\ \downarrow \\ \boxed{\text{III}} \\ = \end{array} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \stackrel{4.3}{=} \left| \begin{array}{ccc} 8 & 16 & 8 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right| \end{array}$$

$$\begin{array}{c} \boxed{\text{II}} \\ = 8 \cdot \left| \begin{array}{ccc} 1 & 2 & 1 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right| \begin{array}{l} \times(-5) \\ \downarrow \\ \boxed{\text{III}} \\ = 8 \cdot \left| \begin{array}{ccc} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 4 & 7 & 4 \end{array} \right| \end{array} \end{array}$$

例

$$\begin{array}{c} \left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\text{I}} = - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\times(-3)} \text{III} = - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\times 1} \text{III} = - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right| \end{array}$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\begin{array}{c} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\times 3} \text{III} = \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\times 2} \text{III} = \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \stackrel{4.3}{=} \left| \begin{array}{ccc} 8 & 16 & 8 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right| \end{array}$$

$$\text{II} = 8 \cdot \left| \begin{array}{ccc} 1 & 2 & 1 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right| \xrightarrow{\times(-5)} \text{III} = 8 \cdot \left| \begin{array}{ccc} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 4 & 7 & 4 \end{array} \right| \xrightarrow{\times(-4)}$$

例

$$\begin{vmatrix} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\text{I}} = - \begin{vmatrix} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\times(-3)} \begin{vmatrix} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\times 1} \begin{vmatrix} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{vmatrix}$$

$$\stackrel{4.3}{=} - \begin{vmatrix} 8 & 4 \\ -1 & 0 \end{vmatrix} = (-1) \cdot 4 = -4.$$

例

$$\begin{vmatrix} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{vmatrix} \xrightarrow{\times 3} \begin{vmatrix} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{vmatrix} \xrightarrow{\times 2} \begin{vmatrix} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{vmatrix} \stackrel{4.3}{=} \begin{vmatrix} 8 & 16 & 8 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 2 & 1 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{vmatrix} \xrightarrow{\times(-5)} \begin{vmatrix} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 4 & 7 & 4 \end{vmatrix} \xrightarrow{\times(-4)} \begin{vmatrix} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 0 & 3 & -12 \end{vmatrix}$$

例

$$\begin{array}{c} \left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \boxed{\text{I}} \\ \updownarrow \\ = - \end{array} \left| \begin{array}{ccc} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \times(-3) \\ \downarrow \\ \boxed{\text{III}} \\ = - \end{array} \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \times 1 \\ \downarrow \\ \boxed{\text{III}} \\ = - \end{array} \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right| \end{array}$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\begin{array}{c} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \begin{array}{l} \times 3 \\ \downarrow \\ \boxed{\text{III}} \\ = \end{array} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \begin{array}{l} \times 2 \\ \downarrow \\ \boxed{\text{III}} \\ = \end{array} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \stackrel{4.3}{=} \left| \begin{array}{ccc} 8 & 16 & 8 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right| \end{array}$$

$$\begin{array}{c} \boxed{\text{II}} \\ = 8 \cdot \left| \begin{array}{ccc} 1 & 2 & 1 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right| \begin{array}{l} \times(-5) \\ \downarrow \\ \boxed{\text{III}} \\ = 8 \cdot \left| \begin{array}{ccc} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 4 & 7 & 4 \end{array} \right| \begin{array}{l} \times(-4) \\ \downarrow \\ \boxed{\text{III}} \\ = 8 \cdot \left| \begin{array}{ccc} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 0 & -1 & 0 \end{array} \right| \end{array} \end{array}$$

例

$$\begin{array}{c} \boxed{\text{I}} \\ \left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \updownarrow \\ = - \\ \downarrow \end{array} \left| \begin{array}{ccc} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \\ \times(-3) \\ \\ \end{array} \boxed{\text{III}} \\ = - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \begin{array}{l} \\ \\ \downarrow \end{array} \times 1 \\ \boxed{\text{III}} \\ = - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right| \end{array}$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\begin{array}{c} \boxed{\text{III}} \\ \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \begin{array}{l} \\ \downarrow \times 3 \\ = \\ \downarrow \end{array} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \begin{array}{l} \\ \\ \times 2 \\ \downarrow \end{array} \boxed{\text{III}} \\ = \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \begin{array}{l} \\ \\ \\ \downarrow \end{array} \stackrel{4.3}{=} \left| \begin{array}{ccc} 8 & 16 & 8 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right| \end{array}$$

$$\begin{array}{c} \boxed{\text{II}} \\ = 8 \cdot \left| \begin{array}{ccc} 1 & 2 & 1 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right| \begin{array}{l} \\ \downarrow \times(-5) \\ \end{array} \boxed{\text{III}} \\ = 8 \cdot \left| \begin{array}{ccc} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 4 & 7 & 4 \end{array} \right| \begin{array}{l} \\ \downarrow \times(-4) \\ \end{array} \boxed{\text{III}} \\ = 8 \cdot \left| \begin{array}{ccc} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 0 & -1 & 0 \end{array} \right| \stackrel{4.3}{=} \end{array}$$

例

$$\begin{array}{c} \left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\text{I}} = - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\times(-3)} \text{III} = - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\times 1} \text{III} = - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right| \end{array}$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\begin{array}{c} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\times 3} \text{III} = \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\times 2} \text{III} = \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \stackrel{4.3}{=} \left| \begin{array}{ccc} 8 & 16 & 8 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right| \end{array}$$

$$\begin{array}{c} \text{II} = 8 \cdot \left| \begin{array}{ccc} 1 & 2 & 1 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right| \xrightarrow{\times(-5)} \text{III} = 8 \cdot \left| \begin{array}{ccc} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 4 & 7 & 4 \end{array} \right| \xrightarrow{\times(-4)} \text{III} = 8 \cdot \left| \begin{array}{ccc} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 0 & -1 & 0 \end{array} \right| \stackrel{4.3}{=} 8 \cdot \left| \begin{array}{cc} -1 & 11 \\ -1 & 0 \end{array} \right| \end{array}$$

例

$$\begin{vmatrix} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\text{I}} = - \begin{vmatrix} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\times(-3)} \begin{vmatrix} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\times 1} \begin{vmatrix} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{vmatrix}$$

$$\stackrel{4.3}{=} - \begin{vmatrix} 8 & 4 \\ -1 & 0 \end{vmatrix} = (-1) \cdot 4 = -4.$$

例

$$\begin{vmatrix} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{vmatrix} \xrightarrow{\times 3} \begin{vmatrix} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{vmatrix} \xrightarrow{\times 2} \begin{vmatrix} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{vmatrix} \stackrel{4.3}{=} \begin{vmatrix} 8 & 16 & 8 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{vmatrix}$$

$$\stackrel{\text{II}}{=} 8 \cdot \begin{vmatrix} 1 & 2 & 1 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{vmatrix} \xrightarrow{\times(-5)} \begin{vmatrix} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 4 & 7 & 4 \end{vmatrix} \xrightarrow{\times(-4)} \begin{vmatrix} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 0 & -1 & 0 \end{vmatrix} \stackrel{4.3}{=} 8 \cdot \begin{vmatrix} -1 & 11 \\ -1 & 0 \end{vmatrix}$$

$$= 8 \cdot 11$$

例

$$\begin{vmatrix} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\text{I}} = - \begin{vmatrix} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\times(-3)} \begin{vmatrix} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\times 1} \begin{vmatrix} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{vmatrix}$$

$$\stackrel{4.3}{=} - \begin{vmatrix} 8 & 4 \\ -1 & 0 \end{vmatrix} = (-1) \cdot 4 = -4.$$

例

$$\begin{vmatrix} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{vmatrix} \xrightarrow{\times 3} \begin{vmatrix} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{vmatrix} \xrightarrow{\times 2} \begin{vmatrix} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{vmatrix} \stackrel{4.3}{=} \begin{vmatrix} 8 & 16 & 8 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{vmatrix}$$

$$\stackrel{\text{II}}{=} 8 \cdot \begin{vmatrix} 1 & 2 & 1 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{vmatrix} \xrightarrow{\times(-5)} \begin{vmatrix} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 4 & 7 & 4 \end{vmatrix} \xrightarrow{\times(-4)} \begin{vmatrix} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 0 & -1 & 0 \end{vmatrix} \stackrel{4.3}{=} 8 \cdot \begin{vmatrix} -1 & 11 \\ -1 & 0 \end{vmatrix}$$

$$= 8 \cdot 11 = 88.$$

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix}$$

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix}$$

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix} = \boxed{\text{III}}$$

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix} \begin{matrix} \uparrow \\ \uparrow \end{matrix} \begin{matrix} \text{III} \\ \text{III} \end{matrix} = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix} = \boxed{\text{III}} \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$\boxed{\text{II}}$

=

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix} = \boxed{\text{III}} \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$$\boxed{\text{II}} = (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ y & x & y \\ z & z & x \end{vmatrix}$$

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix} \begin{matrix} \text{III} \\ \\ \end{matrix} = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$$\begin{matrix} \text{II} \\ \\ \end{matrix} = (x+y+z) \begin{vmatrix} \mathbf{1} & 1 & 1 \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \downarrow \times (-y) \\ \downarrow \times (-z) \end{matrix}$$

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix} \begin{matrix} \text{III} \\ \\ \end{matrix} = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$$\begin{matrix} \text{II} \\ \\ \end{matrix} = (x+y+z) \begin{vmatrix} \mathbf{1} & 1 & 1 \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \downarrow \times (-y) \\ \downarrow \times (-z) \end{matrix} \begin{matrix} \text{III} \\ \\ \end{matrix} =$$

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix} \begin{matrix} \text{III} \\ \\ \end{matrix} = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$$\begin{matrix} \text{II} \\ \\ \end{matrix} = (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \downarrow \times (-y) \\ \downarrow \times (-z) \end{matrix} \begin{matrix} \text{III} \\ \\ \end{matrix} = (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ 0 & x-y & 0 \\ 0 & 0 & x-z \end{vmatrix}$$

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix} \begin{matrix} \text{III} \\ \\ \end{matrix} = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$$\begin{matrix} \text{II} \\ \\ \end{matrix} = (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \downarrow \times (-y) \\ \downarrow \times (-z) \end{matrix} \begin{matrix} \text{III} \\ \\ \end{matrix} = (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ 0 & x-y & 0 \\ 0 & 0 & x-z \end{vmatrix}$$

4.3

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix} \stackrel{\text{III}}{=} \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$$\stackrel{\text{II}}{=} (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \downarrow \times (-y) \\ \downarrow \times (-z) \end{matrix} \stackrel{\text{III}}{=} (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ 0 & x-y & 0 \\ 0 & 0 & x-z \end{vmatrix}$$

$$\stackrel{4.3}{=} (x+y+z) \begin{vmatrix} x-y & 0 \\ 0 & x-z \end{vmatrix}$$

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix} \begin{matrix} \text{III} \\ \\ \end{matrix} = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$$\begin{matrix} \text{II} \\ \\ \end{matrix} = (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \downarrow \times (-y) \\ \downarrow \times (-z) \end{matrix} \begin{matrix} \text{III} \\ \\ \end{matrix} = (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ 0 & x-y & 0 \\ 0 & 0 & x-z \end{vmatrix}$$

$$\stackrel{4.3}{=} (x+y+z) \begin{vmatrix} x-y & 0 \\ 0 & x-z \end{vmatrix} = (x+y+z)(x-y)(x-z).$$

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix} \begin{matrix} \text{III} \\ \\ \end{matrix} = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$$\begin{matrix} \text{II} \\ \\ \end{matrix} = (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \downarrow \times(-y) \\ \downarrow \times(-z) \end{matrix} \begin{matrix} \text{III} \\ \\ \end{matrix} = (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ 0 & x-y & 0 \\ 0 & 0 & x-z \end{vmatrix}$$

$$\stackrel{4.3}{=} (x+y+z) \begin{vmatrix} x-y & 0 \\ 0 & x-z \end{vmatrix} = (x+y+z)(x-y)(x-z).$$

注意

n 次正方行列 A に対して、定理 4.6 を n 回使うと、
 $|kA| = k^n |A|$. $\cdots |kA| \neq k|A|$ ($n \geq 2$) に注意する