

はじめに (数学基礎 B2)

数学基礎 B = 線形代数

教科書 「要点明解 線形数学」 培風館

(第1章 ベクトル)

(第2章 行列)

(第3章 連立1次方程式)

▶ 第4章 行列式

▶ 第5章 行列の対角化

講義の情報 <http://mathweb.sc.niigata-u.ac.jp/~hoshi/teaching-j.html>

シラバス [LINK](#)

- ▶ ノートを取りながら講義を聴くこと。
(ノートを回収して確認する可能性があります)
- ▶ 講義 → 小テスト (理解度確認テスト, 学務情報システム内)

定理 4.6

n 次正方行列 A のある行を k 倍した行列 A'' に対して, $|A''| = k|A|$.

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例

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$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, A'' = \begin{pmatrix} ka & kb \\ c & d \end{pmatrix} \Rightarrow |A''| = kad - kbc = k(ad - bc) = k|A|.$$

定理 4.7

n 次正方行列 A のある行が行ベクトルの和 $\mathbf{a} + \mathbf{a}'$

$$\Rightarrow |A| = \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \mathbf{a} + \mathbf{a}' \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \mathbf{a} \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} + \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \mathbf{a}' \\ a_{n1} & \cdots & a_{nn} \end{vmatrix}.$$

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例

$$\begin{aligned} A &= \begin{pmatrix} a + a' & b + b' \\ c & d \end{pmatrix} \Rightarrow |A| = (a + a')d - (b + b')c \\ &= (ad - bc) + (a'd - b'c) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} a' & b' \\ c & d \end{vmatrix}. \end{aligned}$$

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$$\stackrel{4.5}{=} |A| + k \cdot 0 = |A|.$$

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$$A \rightarrow \cdots \rightarrow \left(\begin{array}{cccc} a & * & \cdots & * \\ 0 & \color{red}{\boxed{B}} & & \\ \vdots & & & \\ 0 & & & \end{array} \right) \text{の形にできれば, (定理 4.3 より)}$$

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$|A|$ を $a \cdot |B|$ を用いて計算できる.

例

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 -1 & 1 & 1 &
 \end{array}
 \begin{array}{c}
 \updownarrow \\
 = -
 \end{array}
 \begin{array}{ccc|c}
 \textcolor{red}{1} & -2 & -1 & \\
 3 & 2 & 1 & \\
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 \end{array}
 \begin{array}{c}
 \downarrow \times (-3) \\
 = -
 \end{array}
 \boxed{\text{III}}
 \begin{array}{ccc|c}
 \textcolor{red}{1} & -2 & -1 & \\
 0 & 8 & 4 & \\
 -1 & 1 & 1 &
 \end{array}$$

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4.3
=

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$$\begin{array}{c}
 \left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\uparrow \\ \text{I}}} = - \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\downarrow \\ \times(-3)}} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\downarrow \\ \times 1}} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right|
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$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

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$$\left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right|$$

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$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\substack{\downarrow \times 3 \\ \text{III}}} \left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\substack{\downarrow \times 2 \\ \text{IV}}}$$

例

$$\left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\updownarrow \\ \text{I}}} \boxed{\text{I}} = - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\downarrow \times (-3)} \boxed{\text{III}} = - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\downarrow \times 1} \boxed{\text{III}} = - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right|$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\downarrow \times 3} \boxed{\text{III}} = \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\downarrow \times 2} \boxed{\text{III}} =$$

例

$$\left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\updownarrow \\ \text{I}}} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\times(-3)} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\times 1} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right|$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\times 3} \left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\times 2} \left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right|$$

例

$$\left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\updownarrow \\ \text{I}}} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\times(-3)} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\times 1} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right|$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\times 3} \left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\times 2} \left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \stackrel{4.3}{=}$$

例

$$\left| \begin{array}{ccc|c} 3 & 2 & 1 & \\ 1 & -2 & -1 & \\ -1 & 1 & 1 & \end{array} \right| \xrightarrow{\substack{\updownarrow \\ \text{I}}} = - \left| \begin{array}{ccc|c} 1 & -2 & -1 & \\ 3 & 2 & 1 & \\ -1 & 1 & 1 & \end{array} \right| \xrightarrow{\downarrow \times (-3)} = - \left| \begin{array}{ccc|c} 1 & -2 & -1 & \\ 0 & 8 & 4 & \\ -1 & 1 & 1 & \end{array} \right| \xrightarrow{\downarrow \times 1} = - \left| \begin{array}{ccc|c} 1 & -2 & -1 & \\ 0 & 8 & 4 & \\ 0 & -1 & 0 & \end{array} \right|$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc|c} 8 & 4 & \\ -1 & 0 & \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\left| \begin{array}{cccc|c} 1 & 2 & 3 & -1 & \\ -3 & 2 & 7 & 11 & \\ 0 & 5 & 9 & 16 & \\ -2 & 0 & 1 & 6 & \end{array} \right| \xrightarrow{\downarrow \times 3} = \left| \begin{array}{cccc|c} 1 & 2 & 3 & -1 & \\ 0 & 8 & 16 & 8 & \\ 0 & 5 & 9 & 16 & \\ -2 & 0 & 1 & 6 & \end{array} \right| \xrightarrow{\downarrow \times 2} = \left| \begin{array}{cccc|c} 1 & 2 & 3 & -1 & \\ 0 & 8 & 16 & 8 & \\ 0 & 5 & 9 & 16 & \\ 0 & 4 & 7 & 4 & \end{array} \right| \stackrel{4.3}{=} \left| \begin{array}{ccc|c} 8 & 16 & 8 & \\ 5 & 9 & 16 & \\ 4 & 7 & 4 & \end{array} \right|$$

例

$$\left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\updownarrow \\ \text{I}}} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\downarrow \\ \times(-3)}} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\downarrow \\ \times 1}} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right|$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\substack{\downarrow \\ \times 3}} \left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\substack{\downarrow \\ \times 2}} \left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \stackrel{4.3}{=} \left| \begin{array}{ccc} \color{red}{8} & \color{red}{16} & \color{red}{8} \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right|$$

$$\begin{array}{|c|} \hline \text{II} \\ \hline \end{array} =$$

例

$$\begin{vmatrix} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\text{I}} = - \begin{vmatrix} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\times(-3)} = - \begin{vmatrix} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\times 1} = - \begin{vmatrix} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{vmatrix}$$

$$\stackrel{4.3}{=} - \begin{vmatrix} 8 & 4 \\ -1 & 0 \end{vmatrix} = (-1) \cdot 4 = -4.$$

例

$$\begin{vmatrix} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{vmatrix} \xrightarrow{\times 3} = \begin{vmatrix} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{vmatrix} \xrightarrow{\times 2} = \begin{vmatrix} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{vmatrix} \stackrel{4.3}{=} \begin{vmatrix} 8 & 16 & 8 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{vmatrix}$$

$$\stackrel{\text{II}}{=} 8 \cdot \begin{vmatrix} 1 & 2 & 1 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{vmatrix}$$

例

$$\begin{vmatrix} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\substack{\updownarrow \\ \boxed{\text{I}}}} = - \begin{vmatrix} \color{red}{1} & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\substack{\downarrow \times (-3) \\ \boxed{\text{III}}}} = - \begin{vmatrix} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{vmatrix} \xrightarrow{\substack{\downarrow \times 1 \\ \boxed{\text{III}}}} = - \begin{vmatrix} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{vmatrix}$$

$$\stackrel{4.3}{=} - \begin{vmatrix} 8 & 4 \\ -1 & 0 \end{vmatrix} = (-1) \cdot 4 = -4.$$

例

$$\begin{vmatrix} \color{red}{1} & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{vmatrix} \xrightarrow{\substack{\downarrow \times 3 \\ \boxed{\text{III}}}} = \begin{vmatrix} \color{red}{1} & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{vmatrix} \xrightarrow{\substack{\downarrow \times 2 \\ \boxed{\text{III}}}} = \begin{vmatrix} \color{red}{1} & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{vmatrix} \stackrel{4.3}{=} \begin{vmatrix} \color{red}{8} & \color{red}{16} & \color{red}{8} \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{vmatrix}$$

$$\boxed{\text{II}} = 8 \cdot \begin{vmatrix} \color{red}{1} & 2 & 1 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{vmatrix} \xrightarrow{\substack{\downarrow \times (-5)}}$$

例

$$\left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\updownarrow \\ \boxed{\text{I}}}} = - \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\downarrow \times (-3)} \boxed{\text{III}} = - \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\downarrow \times 1} \boxed{\text{III}} = - \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right|$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\downarrow \times 3} \boxed{\text{III}} = \left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\downarrow \times 2} \boxed{\text{III}} = \left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \stackrel{4.3}{=} \left| \begin{array}{ccc} \color{red}{8} & \color{red}{16} & \color{red}{8} \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right|$$

$$\boxed{\text{II}} = 8 \cdot \left| \begin{array}{ccc} \color{red}{1} & 2 & 1 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right| \xrightarrow{\downarrow \times (-5)} \boxed{\text{III}} =$$

例

$$\left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\updownarrow \\ \text{I}}} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\downarrow \\ \times(-3)}} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\downarrow \\ \times 1}} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right|$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\substack{\downarrow \\ \times 3}} \left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\substack{\downarrow \\ \times 2}} \left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \stackrel{4.3}{=} \left| \begin{array}{ccc} \color{red}{8} & \color{red}{16} & \color{red}{8} \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right|$$

$$\stackrel{\text{II}}{=} 8 \cdot \left| \begin{array}{cc} \color{red}{1} & 2 \\ 5 & 9 \\ 4 & 7 \end{array} \right| \xrightarrow{\substack{\downarrow \\ \times(-5)}} \stackrel{\text{III}}{=} 8 \cdot \left| \begin{array}{cc} \color{red}{1} & 2 \\ 0 & -1 \\ 4 & 7 \end{array} \right|$$

例

$$\left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\updownarrow \\ \boxed{\text{I}}}} = - \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\downarrow \times (-3)} \boxed{\text{III}} = - \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\downarrow \times 1} \boxed{\text{III}} = - \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right|$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\downarrow \times 3} \boxed{\text{III}} = \left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\downarrow \times 2} \boxed{\text{III}} = \left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \stackrel{4.3}{=} \left| \begin{array}{ccc} \color{red}{8} & \color{red}{16} & \color{red}{8} \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right|$$

$$\boxed{\text{II}} = 8 \cdot \left| \begin{array}{ccc} \color{red}{1} & 2 & 1 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right| \xrightarrow{\downarrow \times (-5)} \boxed{\text{III}} = 8 \cdot \left| \begin{array}{ccc} \color{red}{1} & 2 & 1 \\ 0 & -1 & 11 \\ 4 & 7 & 4 \end{array} \right| \xrightarrow{\downarrow \times (-4)}$$

例

$$\left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\updownarrow \\ \text{I}}} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\downarrow \times (-3) \\ \text{III}}} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\downarrow \times 1 \\ \text{III}}} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right|$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\substack{\downarrow \times 3 \\ \text{III}}} \left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\substack{\downarrow \times 2 \\ \text{III}}} \left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \stackrel{4.3}{=} \left| \begin{array}{ccc} \color{red}{8} & \color{red}{16} & \color{red}{8} \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right|$$

$$\stackrel{\text{II}}{=} 8 \cdot \left| \begin{array}{cc} \color{red}{1} & 2 \\ 5 & 9 \\ 4 & 7 \end{array} \right| \xrightarrow{\substack{\downarrow \times (-5) \\ \text{III}}} \stackrel{\text{III}}{=} 8 \cdot \left| \begin{array}{cc} \color{red}{1} & 2 \\ 0 & -1 \\ 4 & 7 \end{array} \right| \xrightarrow{\substack{\downarrow \times (-4) \\ \text{III}}} \stackrel{\text{III}}{=}$$

例

$$\left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\updownarrow \\ \text{I}}} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\times (-3) \\ \downarrow}} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\times 1 \\ \downarrow}} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right|$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\substack{\times 3 \\ \downarrow}} \left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\substack{\times 2 \\ \downarrow}} \left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \stackrel{4.3}{=} \left| \begin{array}{ccc} \color{red}{8} & \color{red}{16} & \color{red}{8} \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right|$$

$$\stackrel{\text{II}}{=} 8 \cdot \left| \begin{array}{cc} \color{red}{1} & 2 \\ 5 & 9 \\ 4 & 7 \end{array} \right| \xrightarrow{\substack{\times (-5) \\ \downarrow}} \stackrel{\text{III}}{=} 8 \cdot \left| \begin{array}{cc} \color{red}{1} & 2 \\ 0 & -1 \\ 4 & 7 \end{array} \right| \xrightarrow{\substack{\times (-4) \\ \downarrow}} \stackrel{\text{III}}{=} 8 \cdot \left| \begin{array}{cc} \color{red}{1} & 2 \\ 0 & -1 \\ 0 & -1 \end{array} \right|$$

例

$$\left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\updownarrow \\ \text{I}}} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\times (-3) \\ \downarrow}} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\times 1 \\ \downarrow}} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right|$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\substack{\times 3 \\ \downarrow}} \left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\substack{\times 2 \\ \downarrow}} \left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \stackrel{4.3}{=} \left| \begin{array}{ccc} \color{red}{8} & \color{red}{16} & \color{red}{8} \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right|$$

$$\stackrel{\text{II}}{=} 8 \cdot \left| \begin{array}{cc} \color{red}{1} & 2 \\ 5 & 9 \\ 4 & 7 \end{array} \right| \xrightarrow{\substack{\times (-5) \\ \downarrow}} \stackrel{\text{III}}{=} 8 \cdot \left| \begin{array}{cc} \color{red}{1} & 2 \\ 0 & -1 \\ 4 & 7 \end{array} \right| \xrightarrow{\substack{\times (-4) \\ \downarrow}} \stackrel{\text{III}}{=} 8 \cdot \left| \begin{array}{cc} \color{red}{1} & 2 \\ 0 & -1 \\ 0 & -1 \end{array} \right| \stackrel{4.3}{=}$$

例

$$\left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\updownarrow \\ \text{I}}} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\times(-3) \\ \downarrow}} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\times 1 \\ \downarrow}} \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right|$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\substack{\times 3 \\ \downarrow}} \left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\substack{\times 2 \\ \downarrow}} \left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \stackrel{4.3}{=} \left| \begin{array}{ccc} \color{red}{8} & \color{red}{16} & \color{red}{8} \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right|$$

$$\stackrel{\text{II}}{=} 8 \cdot \left| \begin{array}{cc} \color{red}{1} & 2 \\ 5 & 9 \\ 4 & 7 \end{array} \right| \xrightarrow{\substack{\times(-5) \\ \downarrow}} \stackrel{\text{III}}{=} 8 \cdot \left| \begin{array}{cc} \color{red}{1} & 2 \\ 0 & -1 \\ 4 & 7 \end{array} \right| \xrightarrow{\substack{\times(-4) \\ \downarrow}} \stackrel{\text{III}}{=} 8 \cdot \left| \begin{array}{cc} \color{red}{1} & 2 \\ 0 & -1 \\ 0 & -1 \end{array} \right| \stackrel{4.3}{=} 8 \cdot \left| \begin{array}{cc} -1 & 11 \\ -1 & 0 \end{array} \right|$$

例

$$\left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\text{I}} = - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\times(-3)} = - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\times 1} = - \left| \begin{array}{ccc} 1 & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right|$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\times 3} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\times 2} \left| \begin{array}{cccc} 1 & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \stackrel{4.3}{=} \left| \begin{array}{ccc} 8 & 16 & 8 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right|$$

$$\stackrel{\text{II}}{=} 8 \cdot \left| \begin{array}{ccc} 1 & 2 & 1 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right| \xrightarrow{\times(-5)} \stackrel{\text{III}}{=} 8 \cdot \left| \begin{array}{ccc} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 4 & 7 & 4 \end{array} \right| \xrightarrow{\times(-4)} \stackrel{\text{III}}{=} 8 \cdot \left| \begin{array}{ccc} 1 & 2 & 1 \\ 0 & -1 & 11 \\ 0 & -1 & 0 \end{array} \right| \stackrel{4.3}{=} 8 \cdot \left| \begin{array}{cc} -1 & 11 \\ -1 & 0 \end{array} \right|$$

$$= 8 \cdot 11$$

例

$$\left| \begin{array}{ccc} 3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\substack{\updownarrow \\ \boxed{\text{I}}}} = - \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\downarrow \times (-3)} \boxed{\text{III}} = - \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ -1 & 1 & 1 \end{array} \right| \xrightarrow{\downarrow \times 1} \boxed{\text{III}} = - \left| \begin{array}{ccc} \color{red}{1} & -2 & -1 \\ 0 & 8 & 4 \\ 0 & -1 & 0 \end{array} \right|$$

$$\stackrel{4.3}{=} - \left| \begin{array}{cc} 8 & 4 \\ -1 & 0 \end{array} \right| = (-1) \cdot 4 = -4.$$

例

$$\left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ -3 & 2 & 7 & 11 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\downarrow \times 3} \boxed{\text{III}} = \left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ -2 & 0 & 1 & 6 \end{array} \right| \xrightarrow{\downarrow \times 2} \boxed{\text{III}} = \left| \begin{array}{cccc} \color{red}{1} & 2 & 3 & -1 \\ 0 & 8 & 16 & 8 \\ 0 & 5 & 9 & 16 \\ 0 & 4 & 7 & 4 \end{array} \right| \stackrel{4.3}{=} \left| \begin{array}{ccc} \color{red}{8} & \color{red}{16} & \color{red}{8} \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right|$$

$$\boxed{\text{II}} = 8 \cdot \left| \begin{array}{ccc} \color{red}{1} & 2 & 1 \\ 5 & 9 & 16 \\ 4 & 7 & 4 \end{array} \right| \xrightarrow{\downarrow \times (-5)} \boxed{\text{III}} = 8 \cdot \left| \begin{array}{ccc} \color{red}{1} & 2 & 1 \\ 0 & -1 & 11 \\ 4 & 7 & 4 \end{array} \right| \xrightarrow{\downarrow \times (-4)} \boxed{\text{III}} = 8 \cdot \left| \begin{array}{ccc} \color{red}{1} & 2 & 1 \\ 0 & -1 & 11 \\ 0 & -1 & 0 \end{array} \right| \stackrel{4.3}{=} 8 \cdot \left| \begin{array}{cc} -1 & 11 \\ -1 & 0 \end{array} \right|$$

$$= 8 \cdot 11 = 88.$$

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix}$$

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix}$$

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \xrightarrow{\substack{\uparrow \times 1 \\ \uparrow \times 1}} \begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \times 1 = \boxed{\text{III}}$$

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \xrightarrow{\substack{\uparrow \times 1 \\ \uparrow \times 1}} \boxed{\text{III}} = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \xrightarrow{\substack{\uparrow \times 1 \\ \uparrow \times 1}} \boxed{\text{III}} \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$\boxed{\text{II}}$

=

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \xrightarrow{\times 1} \xrightarrow{\times 1} \boxed{\text{III}} \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$$\boxed{\text{II}} = (x+y+z) \begin{vmatrix} \color{red}{1} & 1 & 1 \\ y & x & y \\ z & z & x \end{vmatrix}$$

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \xrightarrow{\begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix}} \begin{matrix} \text{III} \\ = \end{matrix} \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$$\begin{matrix} \text{II} \\ = \end{matrix} (x+y+z) \begin{vmatrix} \textcolor{red}{1} & 1 & 1 \\ y & x & y \\ z & z & x \end{vmatrix} \xrightarrow{\begin{matrix} \downarrow \times (-y) \\ \downarrow \times (-z) \end{matrix}}$$

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \xrightarrow{\begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix}} \begin{matrix} \text{III} \\ = \end{matrix} \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$$\begin{matrix} \text{II} \\ = \end{matrix} (x+y+z) \begin{vmatrix} \textcolor{red}{1} & 1 & 1 \\ y & x & y \\ z & z & x \end{vmatrix} \xrightarrow{\begin{matrix} \downarrow \times (-y) \\ \downarrow \times (-z) \end{matrix}} \begin{matrix} \text{III} \\ = \end{matrix}$$

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \xrightarrow{\begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix}} \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix} \quad \text{III}$$

$$\begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix} \xrightarrow{\begin{matrix} \downarrow \times (-y) \\ \downarrow \times (-z) \end{matrix}} \begin{vmatrix} 1 & 1 & 1 \\ 0 & x-y & 0 \\ 0 & 0 & x-z \end{vmatrix} \quad \text{II} \quad \text{III}$$

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \xrightarrow{\substack{\uparrow \times 1 \\ \uparrow \times 1}} \boxed{\text{III}} \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$$\boxed{\text{II}} = (x+y+z) \begin{vmatrix} \color{red}{1} & 1 & 1 \\ y & x & y \\ z & z & x \end{vmatrix} \xrightarrow{\substack{\downarrow \times (-y) \\ \downarrow \times (-z)}} \boxed{\text{III}} = (x+y+z) \begin{vmatrix} \color{red}{1} & 1 & 1 \\ 0 & x-y & 0 \\ 0 & 0 & x-z \end{vmatrix}$$

4.3

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \xrightarrow{\begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix}} \boxed{\text{III}} = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$$\boxed{\text{II}} = (x+y+z) \begin{vmatrix} \color{red}{1} & 1 & 1 \\ y & x & y \\ z & z & x \end{vmatrix} \xrightarrow{\begin{matrix} \downarrow \times (-y) \\ \downarrow \times (-z) \end{matrix}} \boxed{\text{III}} = (x+y+z) \begin{vmatrix} \color{red}{1} & 1 & 1 \\ 0 & x-y & 0 \\ 0 & 0 & x-z \end{vmatrix}$$

$$\stackrel{4.3}{=} (x+y+z) \begin{vmatrix} x-y & 0 \\ 0 & x-z \end{vmatrix}$$

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \xrightarrow{\begin{matrix} \uparrow \times 1 \\ \uparrow \times 1 \end{matrix}} \boxed{\text{III}} = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$$\boxed{\text{II}} = (x+y+z) \begin{vmatrix} \color{red}{1} & 1 & 1 \\ y & x & y \\ z & z & x \end{vmatrix} \xrightarrow{\begin{matrix} \downarrow \times (-y) \\ \downarrow \times (-z) \end{matrix}} \boxed{\text{III}} = (x+y+z) \begin{vmatrix} \color{red}{1} & 1 & 1 \\ 0 & x-y & 0 \\ 0 & 0 & x-z \end{vmatrix}$$

$$\stackrel{4.3}{=} (x+y+z) \begin{vmatrix} x-y & 0 \\ 0 & x-z \end{vmatrix} = (x+y+z)(x-y)(x-z).$$

例

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \text{ を因数分解せよ.}$$

$$\begin{vmatrix} x & y & z \\ y & x & y \\ z & z & x \end{vmatrix} \xrightarrow{\substack{\uparrow \times 1 \\ \uparrow \times 1}} \boxed{\text{III}} \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ y & x & y \\ z & z & x \end{vmatrix}$$

$$\boxed{\text{II}} = (x+y+z) \begin{vmatrix} \color{red}{1} & 1 & 1 \\ y & x & y \\ z & z & x \end{vmatrix} \xrightarrow{\substack{\downarrow \times (-y) \\ \downarrow \times (-z)}} \boxed{\text{III}} = (x+y+z) \begin{vmatrix} \color{red}{1} & 1 & 1 \\ 0 & x-y & 0 \\ 0 & 0 & x-z \end{vmatrix}$$

$$\stackrel{4.3}{=} (x+y+z) \begin{vmatrix} x-y & 0 \\ 0 & x-z \end{vmatrix} = (x+y+z)(x-y)(x-z).$$

注意

n 次正方行列 A に対して, 定理 4.6 を n 回使うと,
 $|kA| = k^n |A|. \cdots |kA| \neq k|A| \ (n \geq 2)$ に注意する