- Return to RatProbAlgTori
- Return to MultInvField

FlabbyResolution.gap

Definition of M_G

Let G be a finite subgroup of $\mathrm{GL}(n,\mathbb{Z})$. The G-lattice M_G of rank n is defined to be the G-lattice with a $\mathbb{Z}$-basis $\{u_1,\ldots,u_n\}$ on which G acts by $\sigma(u_i)=\sum_{j=1}^n a_{i,j}u_j$ for any $\sigma=[a_{i,j}]\in G$.

Hminus1

```
‣ Hminus1(G)
```

returns the Tate cohomology group $\widehat{H}^{-1}(G,M_G)$ for a finite subgroup $G\leq \mathrm{GL}(n,\mathbb{Z})$.

H0

```
‣ H0(G)
```

returns the Tate cohomology group $\widehat{H}^{0}(G,M_G)$ for a finite subgroup $G\leq \mathrm{GL}(n,\mathbb{Z})$.

H1

```
‣ H1(G)
```

returns the cohomology group $H^1(G,M_G)$ for a finite subgroup $G\leq \mathrm{GL}(n,\mathbb{Z})$.

Z0lattice

```
‣ Z0lattice(G)
```

returns a $\mathbb{Z}$-basis of the group of Tate 0-cocycles $\widehat{Z}^{0}(G,M_G)$ for a finite subgroup $G\leq \mathrm{GL}(n,\mathbb{Z})$.

ConjugacyClassesSubgroups2, ConjugacyClassesSubgroupsFromPerm

```
‣ ConjugacyClassesSubgroups2(G)
```

```
‣ ConjugacyClassesSubgroupsFromPerm(G)
```

returns the list of conjugacy classes of subgroups of a group G. We use this function because the ordering of the conjugacy classes of subgroups of G by the built-in function ConjugacyClassesSubgroups(G) is not fixed for some groups. If a group G is too big, ConjugacyClassesSubgroups2(G) may not work well.

IsFlabby

```
‣ IsFlabby(G)
```

returns whether G-lattice M_G is flabby or not.

IsCoflabby

```
‣ IsCoflabby(G)
```

returns whether G-lattice M_G is coflabby or not.

FlabbyResoluton

```
‣ FlabbyResolution(G)
```

returns a flabby resolution $0 \to M_G \stackrel{\iota}{\to} P \stackrel{\phi}{\to} F \to 0$ of M_G as follows:

```
‣ FlabbyResolution(G).actionP
```

returns the matrix representation of the action of G on P;

```
‣ FlabbyResolution(G).actionF
```

returns the matrix representation of the action of G on F;

```
‣ FlabbyResolution(G).injection
```

returns the matrix which corresponds to the injection $\iota : M_G \to P$;

```
‣ FlabbyResolution(G).surjection
```

returns the matrix which corresponds to the surjection $\phi : P \to F$.

IsInvertibleF

```
‣ IsInvertibleF(G)
```

returns whether $[M_G]^{fl}$ is invertible.

flfl

```
‣ flfl(G)
```

returns the G-lattice E with $[[M_G]^{fl}]^{fl} = [E]$.

PossibilityOfStablyPermutationF

```
‣ PossibilityOfStablyPermutationF(G)
```

returns a basis $\mathcal{L} = \{l_1, \ldots, l_s\}$ of the solution space of the system of linear equations which is obtained by computing some $\mathbb{Z}$-class invariants. Each isomorphism class of irreducible permutation G-lattices corresponds to a conjugacy class of subgroup H of G by $H \leftrightarrow \mathbb{Z}[G/H]$. Let $H_1, \ldots, H_r$ be conjugacy classes of subgroups of G whose ordering corresponds to the GAP function ConjugacyClassesSubgroups2(*G*). Let F be the flabby class of M_G. We assume that F is stably permutation, i.e. for $x_{r+1} = \pm 1$,

$$\left(\bigoplus_{i=1}^{r} \mathbb{Z}[G/H_i]^{\oplus x_i}\right) \oplus F^{\oplus x_{r+1}} \simeq \bigoplus_{i=1}^{r} \mathbb{Z}[G/H_i]^{\oplus y_i}.$$

Define $a_i = x_i - y_i$ and $b_1 = x_{r+1}$. Then we have for $b_1 = \pm 1$,

$$\bigoplus_{i=1}^{r} \mathbb{Z}[G/H_i]^{\oplus a_i} \simeq F^{\oplus(-b_1)}.$$

$[M_G]^{fl} = 0 \Longrightarrow$ there exist $a_1, \ldots, a_r \in \mathbb{Z}$ and $b_1 = \pm 1$ which satisfy the system of linear equations.

PossibilityOfStablyPermutationM

```
‣ PossibilityOfStablyPermutationM(G)
```

returns the same as PossibilityOfStablyPermutationF(*G*) but with respect to M_G instead of F.

Nlist

```
‣ Nlist(l)
```

returns the negative part of the list l.

Plist

```
‣ Plist(l)
```

returns the positive part of the list l.

StablyPermutationFCheck

```
‣ StablyPermutationFCheck(G,L1,L2)
```

returns the matrix P which satisfies $G_1 P = P G_2$ where G_1 (resp. G_2) is the matrix representation group of the action of G on $(\oplus_{i=1}^{r} \mathbb{Z}[G/H_i]^{\oplus a_i}) \oplus F^{\oplus b_1}$

(resp. $(\oplus_{i=1}^{r}\mathbb{Z}[G/H_i]^{\oplus a_i'})\oplus F^{\oplus b_1'}$) with the isomorphism

$$\left(\bigoplus_{i=1}^{r}\mathbb{Z}[G/H_i]^{\oplus a_i}\right)\oplus F^{\oplus b_1}\simeq\left(\bigoplus_{i=1}^{r}\mathbb{Z}[G/H_i]^{\oplus a_i'}\right)\oplus F^{\oplus b_1'}$$

for lists $L_1=[a_1,\ldots,a_r,b_1]$ and $L_2=[a_1',\ldots,a_r',b_1']$, if P exists. If such P does not exist, this returns false.

StablyPermutationMCheck

```
‣ StablyPermutationMCheck(G,L1,L2)
```

returns the same as StablyPermutationFCheck(*G*,*L1*,*L2*) but with respect to M_G instead of F.

StablyPermutationFCheckP

```
‣ StablyPermutationFCheckP(G,L1,L2)
```

returns a basis $\mathcal{P}=\{P_1,\ldots,P_m\}$ of the solution space of $G_1P=PG_2$ where G_1 (resp. G_2) is the matrix representation group of the action of G on $(\oplus_{i=1}^{r}\mathbb{Z}[G/H_i]^{\oplus a_i})\oplus F^{\oplus b_1}$ (resp. $(\oplus_{i=1}^{r}\mathbb{Z}[G/H_i]^{\oplus a_i'})\oplus F^{\oplus b_1'}$) for lists $L_1=[a_1,\ldots,a_r,b_1]$ and $L_2=[a_1',\ldots,a_r',b_1']$, if P exists. If such P does not exist, this returns [ ].

StablyPermutationMCheckP

```
‣ StablyPermutationMCheckP(G,L1,L2)
```

returns the same as StablyPermutationFCheckP(*G*,*L1*,*L2*) but with respect to M_G instead of F.

StablyPermutationFCheckMat

```
‣ StablyPermutationFCheckMat(G,L1,L2,P)
```

returns true if $G_1P=PG_2$ and $\det P=\pm 1$ where G_1 (resp. G_2) is the matrix representation group of the action of G on $(\oplus_{i=1}^{r}\mathbb{Z}[G/H_i]^{\oplus a_i})\oplus F^{\oplus b_1}$ (resp. $(\oplus_{i=1}^{r}\mathbb{Z}[G/H_i]^{\oplus a_i'})\oplus F^{\oplus b_1'}$) for lists $L_1=[a_1,\ldots,a_r,b_1]$ and $L_2=[a_1',\ldots,a_r',b_1']$. If not, this returns false.

StablyPermutationMCheckMat

```
‣ StablyPermutationMCheckMat(G,L1,L2,P)
```

returns the same as StablyPermutationFCheckMat(*G*,*L1*,*L2*,*P*) but with respect to M_G instead of F.

StablyPermutationFCheckGen

```
‣ StablyPermutationFCheckP(G,L1,L2)
```

returns the list $[\mathcal{M}_1, \mathcal{M}_2]$ where $\mathcal{M}_1 = [g_1, \ldots, g_t]$ (resp. $\mathcal{M}_2 = [g'_1, \ldots, g'_t]$) is a list of the generators of G_1 (resp. G_2) which is the matrix representation group of the action of G on $(\oplus_{i=1}^{r} \mathbb{Z}[G/H_i]^{\oplus a_i}) \oplus F^{\oplus b_1}$ (resp. $(\oplus_{i=1}^{r} \mathbb{Z}[G/H_i]^{\oplus a'_i}) \oplus F^{\oplus b'_1}$) for lists $L_1 = [a_1, \ldots, a_r, b_1]$ and $L_2 = [a'_1, \ldots, a'_r, b'_1]$.

StablyPermutationMCheckGen

```
‣ StablyPermutationMCheckGen(G,L1,L2)
```

returns the same as StablyPermutationFCheckGen(*G*,*L1*,*L2*) but with respect to M_G instead of F.

Norm1TorusJ

```
‣ Norm1TorusJ(d,m)
```

returns the Chevalley module $J_{G/H}$ for the m-th transitive subgroup $G = dTm \leq S_d$ of degree d where H is the stabilizer of one of the letters in G.

DirectSumMatrixGroup

```
‣ DirectSumMatrixGroup(l)
```

returns the direct sum of the groups $G_1, \ldots, G_n$ for the list $l = [G_1, \ldots, G_n]$.

DirectProductMatrixGroup

```
‣ DirectProductMatrixGroup(l)
```

returns the direct product of the groups $G_1, \ldots, G_n$ for the list $l = [G_1, \ldots, G_n]$.

References

[HY17] Akinari Hoshi and Aiichi Yamasaki, Rationality problem for algebraic tori, Mem. Amer. Math. Soc. **248** (2017) no. 1176, v+215 pp. AMS Preprint version: arXiv:1210.4525.