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H3nr.gap

Definition of $H^4_{\rm p}(G,\mathbb{Z})$

$$H^4_{\rm p}(G,\mathbb{Z}) := \sum_{\substack{D(H)\lneq H\leq G:\ \text{up to conjugacy}\\ H'<H:\ \text{maximal with}\\ D(H')=D(H)}} {\rm Cores}^G_H({\rm Image}\{H^2(H,\mathbb{Z})^{\otimes 2} \xrightarrow{\cup} H^4(H,\mathbb{Z})\}$$

H4pFromResolution

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‣ H4pFromResolution(RG)
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prints the number of conjugacy subgroups $H \leq G$ with $D(H) \lneq H$ which is the maximal one having the same commutator subgroup $D(H)$, their SmallGroup IDs of GAP and the computing progress rate, and returns the list $L = [l_1, [l_2, l_3]]$ for a free resolution RG of G where l_1 is the abelian invariant of $H^4_{\rm p}(G,\mathbb{Z})$ with respect to Smith normal form, l_2 is the abelian invariants of $H^4(G,\mathbb{Z})$ with respect to Smith normal form and l_3 is generators of $H^4_{\rm p}(G,\mathbb{Z})$ in $H^4(G,\mathbb{Z})$ for a free resolution RG of G.

H4pFromResolution with "H1trivial" option

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‣ H4pFromResolution(RG:H1trivial)
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prints and returns the same as H4pFromResolution(*RG*) but we reduce the number of subgroups $H \leq G$ (see see [HKY20, Section 5])

Definition of $H^4_{\rm nr}(G,\mathbb{Z})$

$$H^4_{\rm nr}(G,\mathbb{Z}) := \bigcap_{\substack{H\leq G\\ g\in Z_G(H)}} {\rm Ker}(\tilde{\partial}_{H,g})$$

where

$\widetilde{m}^*$ and $\tilde{\partial}$ are given in [Pe08, Definition 5] or [HKY20, Definition 2.11].

IsUnramifiedH3

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‣ IsUnramifiedH3(RG,L)
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prints the number of pairs (H,I) of $H \leq G$ and $I \leq Z(H)$ which satisfy the following conditions (i)-(iv), the computing progress rate and the list $L' = [l_1, l_2]$ where l_1 is the abelian invariant of $H^3(H,\mathbb{Z}) \simeq H^2(H,\mathbb{Q}/\mathbb{Z})$ with respect to Smith normal form and l_2 is the generator of $\tilde{\partial}_{H,g}(L)$ in $H^3(H,\mathbb{Z})$ and returns true (resp. false) if the generator L is in $H^4_{\rm nr}(G,\mathbb{Z})$ (resp. is not in $H^4_{\rm nr}(G,\mathbb{Z})$) for a free resolution RG of G and a generator L of $H^4(G,\mathbb{Z})$:
(i) $I = \langle g \rangle$ for some g;
(ii) (H,I) is chosen up to conjugation;
(iii) $(H',I') \leq (H,I)$ is maximal, and thus we may assume that $H = Z_G(I)$ (where $I = \langle g \rangle$) and g belongs to the center of H;
(iv) $H^3(H,\mathbb{Z}) \neq 0$.

IsUnramifiedH3 with "Subgroup" option

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‣ IsUnramifiedH3(RG,L:Subgroup)
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prints and returns the same as IsUnramifiedH3(*RG*,*L*) but we require the additional condition:
(v) $H^4_{\rm p}(H,\mathbb{Z}) \lneq H^4(H,\mathbb{Z})$.

References

[HKY20] Akinari Hoshi, Ming-chang Kang and Aiichi Yamasaki, Degree three unramified cohomology groups and Noether's problem for groups of order 243, Journal of Algebra **544** (2020) 262-301. ScienceDirect Extended version: arXiv:1710.01958.
[Pe08] Emmanuel Peyre, Unramified cohomology of degree 3 and Noether's problem, Invent. Math. **171** (2008) 191-225.