Niigata Algebra Symposium

December 6th to December 8th, 2023

Department of Mathematics, Faculty of Science, Niigata University, Niigata, Japan

	9:45–10:45	11:00–12:00	14:00–15:00	15:15–16:15	16:30–17:30
6th	A. Hoshi	S. Tanimoto	T. Sano	K. Furukawa	T. Kishimoto
7th	M. Enokizono	T. Katsura	Y. Gongyo	S. Okawa	S. Mori
8th	S. Yoshikawa	T. Terasoma	K. Oguiso	H. Tokunaga	H. Kaji

Titles and Abstracts

Akinari Hoshi (Niigata University)

Birational classification for algebraic tori

We give a stably birational classification for algebraic k-tori of dimensions 3 and 4 over a field k. Kunyavskii [Kun90] proved that there exist 15 not stably k-rational cases among 73 cases of algebraic k-tori of dimension 3. Hoshi and Yamasaki [HY17] showed that there exist exactly 487 (resp. 7, resp. 216) stably k-rational (resp. not stably but retract k-rational, resp. not retract k-rational) cases of algebraic k-tori of dimension 4. First, we define the weak stably k-equivalence of algebraic k-tori and show that there exist 13 (resp. 128) weak stably k-equivalent classes of algebraic k-tori T of dimension 3 (resp. 4) which are not stably k-rational by computing some cohomological stably birational invariants, e.g. the Brauer-Grothendieck group of X where X is a smooth k-compactification of T, provided by Kunyavskii, Skorobogatov and Tsfasman [KST89]. We make a procedure to compute such stably birational invariants effectively and the computations are done by using the computer algebra system GAP. Second, we define the p-part of the flabby class $[\widehat{T}]^{fl}$ as a $\mathbb{Z}_p[Sy_p(G)]$-lattice and prove that they are faithful and indecomposable $\mathbb{Z}_p[Sy_p(G)]$-lattices unless it vanishes for $p = 2$ (resp. $p = 2, 3$) in dimension 3 (resp. 4). The $\mathbb{Z}_p$-ranks of them are also given. Third, we give a necessary and sufficient condition for which two not stably k-rational algebraic k-tori T and T' of dimensions 3 (resp. 4) are stably birationally k-equivalent in terms of the splitting fields and the weak stably k-equivalent classes of T and T'. In particular, the splitting fields of them should coincide if $\widehat{T}$ and $\widehat{T}'$ are indecomposable. Forth, for each 7 cases of not stably but retract k-rational algebraic k-tori of dimension 4, we find an algebraic k-torus T' of dimension 4 which satisfies that $T \times_k T'$ is stably k-rational. Finally, we give a criteria to determine whether two algebraic k-tori T and T' of general dimensions are stably birationally k-equivalent when T (resp. T') is stably birationally k-equivalent to some algebraic k-torus T'' of dimension up to 4. This is a joint work with Aiichi Yamasaki.

Sho Tanimoto (Nagoya University)

Sections of Fano fibrations over curves

Manin's conjecture predicts the asymptotic formula for the counting function of rational points on a smooth Fano variety, and it predicts an explicit asymptotic formula in terms of geometric invariants of the underlying variety. When you count rational points, it is important to exclude some contribution of rational points from an exceptional set so that the asymptotic formula reflects global geometry of the underlying variety. I will discuss applications of the study of exceptional sets to moduli spaces of sections of Fano fibrations, and in particular I will explain how exceptional sets explain pathological components of the moduli space of sections. If time permits, I will discuss the case of del Pezzo fibrations in more details. This is based on joint work with Brian Lehmann and Eric Riedl.

Taro Sano (Kobe University)

Infinitely many families of Sasaki-Einstein metrics on spheres

It is an interesting problem to find Einstein metrics on a given manifold. One can consider the problem on spheres as a starting case. Not many Einstein metrics on spheres were found until Boyer-Galicki-Kollár found (finitely) many Sasaki-Einstein metrics on odd dimensional spheres. Their method is to relate such metrics with Kähler-Einstein metrics on hypersurfaces in complex weighted projective spaces. Recently, Collins-Szélelyhidi found infinitely many families of Sasaki-Einstein metrics on S^5. In this talk, I will talk about how to find infinitely many families of Sasaki-Einstein metrics on odd dimensional spheres by generalizing their methods. This is joint work with Yuchen Liu and Luca Tasin.

Katsuhisa Furukawa (Josai University)

Singular loci and equations of higher secant varieties of Veronese embeddings

For a non-degenerate projective variety X in $\mathbb{P}^N$, we call the closure of the union of all the $(k-1)$-planes spanned by k points on X the k-secant variety of X. When the k-secant variety of X is not equal to the whole space $\mathbb{P}^N$, its singular locus contains the $(k-1)$-secant variety of X. A question then arise, what is the difference between them. I will talk about singularity and non-singularity of the k-secant variety of X in $\mathbb{P}^N$ if X is the image of the d-uple Veronese embedding of $\mathbb{P}^n$, where $N = \binom{n+d}{n} - 1$. In some specific cases, in order to calculate the singular loci, it is important to know defining equations of k-secant varieties explicitly. I will explain minimal and almost minimal degrees of k-secant varieties, and discuss how to use prolongations and representations with respect to Veronese embeddings for the purpose. This is a joint work with Kangjin Han.

Takashi Kishimoto (Saitama University)

Birational Geometry and Cylindricity of Severi-Brauer varieties

A Severi-Brauer n-fold X over a field k is by definition an algebraic variety defined over k whose base extension X_{k_s} to a separable closure k_s of k is isomorphic to $\mathbb{P}^n_{k_s}$. These varieties appear naturally even if we work over an algebraically closed field, e.g., the generic fiber of a $\mathbb{P}^n$-fibration $\pi : V \to W$ over $\mathbb{C}$ is a Severi-Brauer n-fold over $\mathbb{C}(W)$. In order to understand well the birational geometry and cylindricity of the total space V associated to π, those of the generic fiber of π play an important role. In the talk, we will mention at first somehow rough birational properties of Severi-Brauer n-folds depending on the parity of n. Then we focus on the case of $n = 3$, i.e., Severi-Brauer threefolds, and we will look into birational specificities in more detail in addition to the cylindricity/non-cylindricity of such varieties. This is a joint work in progress with Adrien Dubouloz, Kento Fujita and Takuzo Okada.

Makoto Enokizono (Rikkyo University)

Slope inequality of fibered surfaces, Morsification conjecture and moduli of curves

Slope inequalities of fibered surfaces play an important role in the classification problem of algebraic surfaces of general type. Morsification conjecture predicts that any fiber germ of fibered surfaces has a splitting deformation to a finite number of stable fiber germs with one node and smooth multiple fiber germs, which are solved in the case of reduced fiber germs and the small genus case. In this talk, I will explain that under the assumption of the Morsification conjecture, many slope inequalities of "general" fibered surfaces can be obtained by using the theory of moduli of curves. If time permits, I will also explain some applications, related topics and problems.

Toshiyuki Katsura (The University of Tokyo)

On the classification of Enriques surfaces with finite automorphism group

In 1980s, Kondo classified Enriques surfaces with finite automorphism group over the field of complex numbers. He also determined the structure of automorphism group and the number of moduli for each class. In the late 2010s, Martin carried out similar results to the one by Kondo over an algebraically closed field of characteristic $p > 2$. Then, Kondo, Martin and the speaker gave the classification of Enriques surfaces with finite automorphism group in characteristic 2. However, they didn't determine the structure of automorphism group and the number of moduli in this case. As the final step of this classification we solve these two left problems. This is a joint-work with Matthias Schuett.

Yoshinori Gongyo (The University of Tokyo)

The Mukai type conjecture

I introduce the total invariant for Fano manifolds and propose the new conjecture for the characterization of the product of projective space. And I will explain the relation with the original Mukai conjecture due to the picard number and the Fano index.

Shinnosuke Okawa (Osaka University)

Blowing down noncommutative cubic surfaces

In 2001 Van den Bergh defined the notion of blowup for noncommutative surfaces and proved that the blowup of a noncommutative $\mathbb{P}^2$ in 6 points is isomorphic to a cubic hypersurface of a noncommutative $\mathbb{P}^3$. In this talk, based on a joint work with Ingalls, Sierra, and Van den Bergh, I will show that the converse holds as well; namely, cubic hypersurfaces of noncommutative $\mathbb{P}^3$s are isomorphic to blowups of noncommutative $\mathbb{P}^2$s under reasonable assumptions. One of the key ingredients of the argument is to understand the monodromy of lines on noncommutative cubic surfaces. I would also like to explain how it is helpful to consider the Poisson geometry obtained as the semi-classical limit.

Shigefumi Mori (Kyoto University)

Three-dimensional extremal contractions with 1-dimensional fibers

The Minimal Model Program (MMP) has been developed in all dimensions to accommodate many applications. In dimension 3, the MMP has been made especially explicit in many cases. In this talk, I will survey some of the explicit results on the 3-dimensional extremal contractions with possibly reducible 1-dimensional fibers over the reference point.

Shou Yoshikawa (Tokyo Institute of Technology)

Characterization of toric pair via log tangent bundle in positive characteristic

Toric pair is a pair of a toric variety and the sum of torus-invariant prime divisors. In characteristic zero, toric pairs are characterized by the triviality of the log tangent bundle for rationally connected varieties due to Winkelmann. In positive characteristic, a similar result is known in the case where the given variety is globally Frobenius-split by Achinger Witaszek Zdanowicz. In this talk, I will introduce two analogs of Achinger-Witaszek-Zdanowicz ' s result. First one is replacing the triviality by some numerical condition, and the second one is removing globally Frobenius splitting if the variety is separablly rationally connected. This talk is based on joint work with Sho Ejiri.

Tomohide Terasoma (Hosei University)

Depth filtration of MZV and degenerateion of elliptic curves

On the space of multiple zeta values (MZV) one can define the depth filtration, which is supposed to related to the space of cusp forms. The celebrated conjecture by Broadhurst-Kreimer suggests the relation between the relative completion of fundamental group of moduli space of elliptic curves and mixed Tate motives. They are related via degeneration of universal elliptic curves. In this talk, we try to explain the exact meaning of the relation between these two objects.

Keiji Oguiso (The University of Tokyo)

The Kawaguchi-Silverman Conjecture for endomorphisms of Ueno-type varieties

Ueno-type varieties are the blow-up of the cyclic quotient varieties $E^n/\langle \zeta_n I_n \rangle$ of the self-product E^n of an elliptic curve E at the singular points, where ζ_n is a primitive n-th root of 1 with $n = 2, 3, 4$ or 6 according to the types of E. Ueno-type varieties provide interesting examples in birational geometry, algebraic dynamics and their interactions as observed by several authors. In this talk, after reviewing some of above-mentioned phenomena and open problems, I would like to show the Kawaguchi-Silverman Conjecture (KSC) for endomorphisms of Ueno-type varieties defined over $\bar{\mathbb{Q}}$. This provides a first non-trivial and non-vacuous example of rationally connected varieties of dimension 3, 4, 5 with no int-amplified endomorphism, satisfying KSC for all endomorphisms.

Hiro-o Tokunaga (Tokyo Metropolitan University)

Arithmetic of double covers of $\mathbb{P}^2$ and conic-line arrangements

Let B and C be reduced (possibly reducible) plane curves with no common components. Assume that $\deg B$ is even and consider a double cover $f'_B : S'_B \to \mathbb{P}^2$ branched along B and let $\mu_B : S_B \to S'_B$ be the canonical resolution of singularities. Put $\tilde{f}_B := f'_B \circ \mu_B$. The preimage $\tilde{f}_B^* C$ gives a divisor on S_B. We investigate properties of $\tilde{f}_B^* C$ and apply our consideration to study the embedded topology of $B + C$. To this purpose, we choose a smooth point z_o on B and we associate a surface S_{B,z_o} having a hyperellitpic/an elliptic fibration induced by z_o. Under these settings, arithmetic properties of the generic fiber of S_{B,z_o} play important roles to study $\tilde{f}_B^* C$. We apply this consideration to study the topology of conic-line arrangements.

Hajime Kaji (Waseda University)

Some remarks on degree formulae for 2-step flag varieties

Two degree formulae for 2-step flag varieties are presented: One is an easy application of Kaji-Terasoma's Gysin formula for Grassmann bundles; the other is a simple computation by Borel-Hirzebruch's degree formula for rational homogeneous varieties. Then, some observations are given through comparison between them.

Organizers: Kenta Hashizume, Akinari Hoshi, Hideo Kojima, Takuzo Okada, Kazuhiko Yamaki.