The 2nd Niigata Algebra Symposium

November 27th to November 30th, 2024

Department of Mathematics, Faculty of Science, Niigata University, Niigata, Japan

	9:40–10:40	11:00–12:00	14:00–15:00	15:20–16:20	16:40–17:40
27th			K. Hashizume	Y. Matsuzawa	O. Fujino
28th	K. Shimomoto	M. Kida	T. Yasuda	T. Ashikaga	A. Moriwaki
29th	Y. Nakamura	K. Watanabe	R. Takahashi	K. Konno	S. Mukai
30th	P. A. Helminck	M. Nagaoka	T. Koike	H. Ohashi	A. Takahashi

Titles and Abstracts

Kenta Hashizume (Niigata University)

On boundedness and moduli space of special klt-trivial fibrations

A klt-trivial fibration is a kind of fibration which often appears in birational geometry. In this talk, I will introduce the boundedness result and the existence of the coarse moduli space of special klt-trivial fibrations over curves. I will mainly explain the boundedness of the special klt-trivial fibrations over curves with some fixed invariants. This talk is based on a joint work with Masafumi Hattori.

Yohsuke Matsuzawa (Osaka Metropolitan University)

Height growth along orbits and Zariski dense orbit conjecture

For a dominant rational self-map on a projective variety defined over $\overline{Q}$, the exponential growth rate of height along orbits is called arithmetic degree. It is defined as a limit of the form $\lim_{n\to\infty} h(f^n(x))^{1/n}$, where h is a Weil height function. For any orbit, arithmetic degree is bounded above by the first dynamical degree of the map, and it is conjectured that the equality holds for Zariski dense orbit. In general, the existence of the limit is not known, but we recently proved that the limit exists for generic orbit, orbit that is infinite and intersects with proper closed subset only finitely many times. Moreover, we also proved that there are many points whose arithmetic degree exists and arbitrary close to the first dynamical degree. We applied this results to construct points that has Zariski dense orbits, which solves some special cases of Zariski dense orbit conjecture for birational maps. Part of the work is based on joint work with Junyi Xie.

Osamu Fujino (Kyoto University)

On finiteness of relative log pluricanonical representations

We explain the finiteness of relative log pluricanonical representations in a suitable complex analytic setting. It fills a missing part of the minimal model program for projective morphisms between complex analytic spaces. By combining it with some recent results obtained by Enokizono and Hashizume, we can prove the existence of log canonical flips for complex analytic spaces, and so on.

Kazuma Shimomoto (Tokyo Institute of Technology)

Frobenius liftablity and etale quotients of projective varieties

The classical Serre-Tate's theory asserts that an ordinary Abelian variety in positive characteristic has a canonical lift with Frobenius over the Witt vectors. This was later generalized by Nori and Srinivas for a wider class of projective varieties. In this talk, we examine this problem in the case of finite etale quotients of Frobenius-liftable varieties. This is joint work with R. Ishizuka.

Masanari Kida (Tokyo University of Science)

Variations on the theme of arithmetic equivalence

Two number fields (we assume that they are of finite degree over the rationals) are said to be arithmetically equivalent if their Dedekind zeta functions coincide. There are many known examples of non-conjugate arithmetically equivalent fields, and similar constructions are also found in areas like differential geometry and graph theory. The talk will explore various generalizations of this concept in the context of number fields using Artin L-functions etc. This work is partly a joint work with Yuta Katayama.

Takehiko Yasuda (Osaka University)

Motivic mass formulas and permutation quotient singularities

Krasner, Serre and Bhargava showed mass formulas for counting extensions of local fields. As an application, we can prove that quotient singularities by permutation actions are always canonical in any characteristic. This strengthens the previously known fact that such singularities are log canonical, as a consequence of the fact that such singularities are F-pure, which was observed by Hochster and Huneke.

Tadashi Ashikaga (Tohoku Gakuin University)

Moduli maps near hyperelliptic singular fibers in non-hyperelliptic fibrations of genus 3

Let $X(f)$ be an invariant of a fibered complex surface $f : S \to B$ of genus g. Our general motivation is to "pulling back $X(f)$ via moduli map" explicitly from the corresponding $\mathbf{Q}$-divisor D_X on $\overline{M}_g$. If we choose D_X as the Harris-Mumford divisor (consisting of non-maximal-gonal curves) for odd genus, then this process is closely related to the determination of Horikawa index of f. Today, we discuss with a non-hyperelliptic fibration f of genus 3 and $D_X = \overline{H}_3$ is the hyperelliptic divisor on $\overline{M}_3$. Our goal is to classify all the "hyperelliptic singular fibers in f" quantitatively by using orbifold intersection numbers.

Atsushi Moriwaki (Chubu University)

Adelic structures of countable fields

This is a joint work with Huayi Chen. The most important target for the theory of adelic curves is a variety over a countable field of characteristic zero. In this talk, I would like to give the standard way to adelic structures of such fields. As applications, I will propose a generalized Fermat conjecture for a finitely generated field over the rational number field.

Yusuke Nakamura (Nagoya University)

A counterexample to the PIA conjecture

In this talk, I will give a counterexample to the PIA (precise inversion of adjunction) conjecture for MLD's (minimal log discrepancy). The usual inversion of adjunction is a type of claim "the information of the singularity of a pair (X,D) can be recovered from the information of the singularity of D". The precise version (PIA conjecture) states that this is correct at the level of MLD (minimal log discrepancy), the invariant of the singularity. The PIA conjecture is known to be true in dimension 3. In this talk, I will give a counterexample in dimension 5. This talk is based on joint work with Kohsuke Shibata.

Kiwamu Watanabe (Chuo University)

Fano varieties with large pseudoindex

We give a classification of smooth Fano varieties X with Picard number greater than one when the pseudoindex is large.

Ryo Takahashi (Nagoya University)

Generating Cohen–Macaulay module categories and Buchweitz–Orlov singularity categories

Cohen–Macaulay representation theory was born in the 1960s as a higher-dimensional version of representation theory of artin algebras. The purpose of this theory is, for a given Cohen–Macaulay ring R, to understand the structure of the category $\mathrm{CM}(R)$ of (maximal) Cohen–Macaulay R-modules. In this talk, we will first consider a certain question on how to generate the category $\mathrm{CM}(R)$ by using concrete examples. Next we will move on to thinking of a similar question for the singularity category $\mathrm{D_{sg}}(R)$ in the sense of Buchweitz and Orlov, and extend theorems of Ballard, Favero and Katzarkov.

Kazuhiro Konno (Kansai University)

Decompositions of elliptic cycles

In the study of normal surface singularities, it is sometimes important and helpful for intuitive understanding to have the resolution dual graphs of the exceptional sets. Listing all the possible configurations, however, requires huge efforts even in a restricted situation and sometimes not practical. The purpose of the talk is to announce a small contribution to weakly elliptic singularities. We describe the decomposition of the fundamental cycle minus its associated minimally elliptic cycle, that sketches out what happens along Yau's elliptic sequence.

Shigeru Mukai (Kyoto University)

Cubic 4-folds with eleven cusps and their applications

We construct a cubic 4-fold whose singularities are 11 cusps and which has an action of the Mathieu group M_{11}, all over the ternary field $\mathbb{F}_3$. As an application we partly answer a question of Shimada-Zhang concerning the description of $\mathrm{K3}^{ss1}(p=3)$, that is, supersingular K3 surfaces of Artin invariant one in characteristic 3, as inseparable triple quadric. As another application, we ask two questions: i) on a moduli of bundles on this $\mathrm{K3}^{ss1}$ with McLaughlin configuration of 275 (-2) Mukai vectors, and (ii) on its relation with M_{11}.

Paul Alexander Helminck (Tohoku University)

Finding pair-of-pants decompositions of algebraic varieties using tropical geometry

In this talk, I will show how one can use tropical geometry to construct an explicit CW-complex that realizes the homotopy type of an algebraic variety. This technique conjecturally works for all algebraic varieties, and it generalizes the well-known pair-of-pants decomposition for compact Riemann surfaces. The main ingredients in the construction are:

-Toric degenerations of complements of hyperplane arrangements,

-Kato-Nakayama spaces, and

-Smooth tropicalizations.

I will show how all of these enter the picture, and how they work in practice for various varieties, including K3-surfaces. This is joint work with Yassine El Maazouz.

Masaru Nagaoka (Gakushuin University)

Completion of the affine 3-space into sextic del Pezzo fibrations

Yu. G. Prokhorov constructed examples of completions of the affine 3- fold $\mathbb{A}^3$ into smooth 3-folds endowed with sextic del Pezzo fibrations in his study on singular Fano 3-folds of genus 12. By construction, the boundary divisor consists of a fiber and a horizontal divisor which is non-normal along a section. In this talk, we explain how to construct new examples of completions of $\mathbb{A}^3$ into smooth 3-folds endowed with sextic del Pezzo fibrations, whose horizontal divisors are normal along any sections.

Takayuki Koike (Osaka Metropolitan University)

Formal principle for line bundles on neighborhoods of an analytic subset of a compact Kähler manifold

We investigate the formal principle for holomorphic line bundles on neighborhoods of an analytic subset of a complex manifold mainly when it can be realized as an open subset of a compact Kähler manifold.

Hisanori Ohashi (Tokyo University of Science)

Finite groups of symplectic automorphisms of the superspecial K3 surface in positive characteristic

In 1988, Mukai classified the finite groups of symplectic automorphisms acting on complex K3 surfaces. Later in 2000s, Dolgachev and Keum extended the result toward K3 surfaces in positive characteristics, mainly in tame cases. In a joint work with M. Schuett (Hannover), we will give a complete classification for the superspecial K3 surface in positive characteristics. We expect that the classification of finite groups remains true even when we consider all K3 surfaces, but this is still conjectural.

Atsushi Takahashi (Osaka University)

Maximally-graded matrix factorizations and the Gamma integral structure for an invertible polynomial of chain type

An invertible polynomial of chain type is a weighted homogeneous polynomial having nice combinatorial properties. Motivated by the Orlik-Randell conjecture and the homological mirror symmetry conjecture, we construct a full exceptional collection in the category of maximally-graded matrix factorizations. As an application, we show that the Gamma integral structure defined through the full exceptional collection is isomorphic to the natural one from the homology group of the Milnor fiber of its mirror dual. This talk is based on my joint work with Daisuke Aramaki and the one with Takumi Otani.

Organizers: Kenta Hashizume, Akinari Hoshi, Hideo Kojima, Takuzo Okada, Kazuhiko Yamaki.