The 3rd Niigata Algebra Symposium

November 19th to November 22nd, 2025

Department of Mathematics, Faculty of Science, Niigata University, Niigata, Japan

	9:40–10:40	11:00–12:00	14:00–15:00	15:20–16:20	16:40–17:40
19th	(Free)	(Free)	Ito	Kuroda	Katsura
20th	Miyanishi	Iwai	Ma	Kondo	Kawamata
21st	Yamasaki	Yoshinaga	Iyama	Yasufuku	Tamagawa
22nd	Kawakami	Takagi	Ishii	Fujita	Namikawa

Titles and Abstracts

Hiroyuki Ito (Tokyo University of Science)

Quasi-fibration on rational surfaces

Due to the failure of Bertini's theorem, there exist algebraic surfaces in positive characteristic with a fibration structure whose general fibers are singular. In such fibrations, the arithmetic genus of the generic fiber differs from the genus of normalization of the generated fiber, and the genus change formula by Tate restricts the characteristic depending on the genus of the fiber. A well-known example is the quasi-elliptic surface, which exists only in characteristics 2 and 3. As a "degenerate" elliptic surface, it played an important role in Enriques' classification of surfaces by Bombieri-Mumford. In this talk, I will discuss the classification of singular fibers of quasi-hyperelliptic surfaces, which are higher genus versions of quasi-elliptic surfaces, and classify the combination of singular fibers for rational surfaces. If time permits, I will also cover more general quasi-fibrations.

Shigeru Kuroda (Tokyo Metropolitan University)

Nagata's conjecture on a polynomial automorphism over a field of positive characteristic

An automorphism of the polynomial ring in n variables over a field k is said to be tame if it is obtained by composing affine automorphisms and elementary automorphisms. Jung and van der Kulk showed that every automorphism is tame if $n = 2$ around 1950. In 1972, Nagata conjectured that there exists a non-tame automorphism when $n = 3$, and constructed a candidate. In 2003, Shestakov and Umirbaev settled this conjecture in the affirmative for $\mathrm{char}(k) = 0$. Recently, I proved that Nagata's conjecture is true if $\mathrm{char}(k)$ is seven and above. This is the first time that the existence of a non-tame automorphism has been confirmed in the positive characteristic case. In this talk, I will explain the proof strategy and the key inequality used in the proof.

Toshiyuki Katsura (The University of Tokyo)

The 2-divisibility of divisors on K3 surfaces

Let X be a K3 surface defined over an algebraically closed field k, and let E_i $(i = 1, 2, \ldots, n)$ be pairwise disjoint (-2)-curves. If the divisor $\sum_{i=1}^{n} E_i$ is divisible by 2 in $\mathrm{Pic}(X)$, then it defines a double covering of X, yielding a new surface. In characteristic $p = 0$, Nikulin showed that if $\sum_{i=1}^{n} E_i$ is divisible by 2 in $\mathrm{Pic}(X)$, n must be either 8 or 16. Moreover, when $n = 16$, the covering A is an abelian surface, and X is the associated Kummer surface. These results also remain valid in positive characteristic $p \neq 2$. However, when $p = 2$, the covering is purely inseparable, and the situation is completely different. In this talk we show that

in characteristic 2, a K3 surface can contain a set of n disjoint smooth rational curves whose sum is divisible by 2 in the Picard group, for $n = 8, 12, 16, 20$. More precisely, all these values occur on supersingular K3 surfaces, with the exception of Artin invariants 1 and 10, whereas on K3 surfaces of finite height, only $n = 8$ can occur. Finally, we determine the structure of the K3 surface in the case $n = 20$.

Masayoshi Miyanishi (Kwansei Gakuin University)

Del Pezzo surfaces and beyond

Del Pezzo surfaces are classically and well understood. But readers interest tends to head for the finiteness features of (-1)-curves, Weyl groups of reflections, etc. and research in the case 9 points or more of $\mathbb{P}^2$ in general position are blown up is more or less neglected except for the fact that there are infinitely many (-1)-curves on the blown-up surfaces. But there exists interesting geometry like Coble surfaces and Mohan Kumar-Murthy's result which led to the positive solution of Coolidge-Nagata conjecture due to Koras-Palka on cuspidal curves being transformed to a line by a Cremona transformation. As a royal object of geometry, I would like to revive again the interest toward research of rational surfaces obtained by blowing up many points of $\mathbb{P}^2$.

Masataka Iwai (The University of Osaka)

Inequalities for the second Chern class and structure theorems

It was shown by Miyaoka and Yau that for any n-dimensional smooth projective complex variety X with an ample canonical divisor, the Miyaoka-Yau inequality $2(n+1)c_2(X)c_1(X)^{n-2} \geq nc_1(X)^n$ holds. Moreover, if the equality holds in this inequality, then the universal cover of X is isomorphic to the unit ball in $\mathbb{C}^n$. In this talk, I will present several inequalities involving the second Chern class for projective klt varieties and the structure theorems when the equality holds. This talk is based on joint work with Shin-ichi Matsumura (Tohoku University) and Niklas Muller (Universitat Freiburg).

Shouhei Ma (Institute of Science Tokyo)

Siegel modular forms arising from higher Chow cycles

I explain that the infinitesimal invariant of a higher Chow cycle of type $(2, 3-g)$ on a generic abelian variety of dimension $g < 4$ gives rise to a meromorphic Siegel modular form of (virtual) weight $\mathrm{Sym}^4 \otimes \det^{-1}$ with bounded singularity, and this construction is functorial with respect to rank 1 degeneration. The theory of modular forms and that of algebraic cycles have a common origin in the study of Lemniscate; even at the modern stage, these two branches are still connected.

Shigeyuki Kondo (Nagoya University)

Congruences of bitangent lines of Kummer quartic surfaces in any characteristic

A surface in the Grassmannian $G(1, 3)$ of lines in $\mathbb{P}^3$ is classically called a congruence of lines. E. Kummer (1866) discovered that the congruence of bitangent lines of a Kummer quartic surface has 22 (= 6+16) irreducible components. In this talk, we shall discuss Kummer quartic surfaces in any characteristic from the point of view of congruences of bitangent lines. This is a joint work with Igor Dolgachev.

Yujiro Kawamata (The University of Tokyo)

A deformation of a coherent sheaf over a non-commutative base

When we consider a deformation of a sheaf, it is natural not to assume that the parameter ring is commutative. The existence of a versal formal NC deformation is easily proved, but it is difficult to describe it for a specific problem. I will review some cases where versal formal NC deformations are known. I will also talk about a global moduli space.

Aiichi Yamasaki (Kyoto University)

Birational classification for algebraic tori

We give a stably birational classification for algebraic k-tori of dimensions 3 and 4 over a field k. Kunyavskii [Kun90] proved that there exist 15 not stably k-rational cases among 73 cases of algebraic k-tori of dimension 3. Hoshi and Yamasaki [HY17] showed that there exist exactly 487 (resp. 7, resp. 216) stably k-rational (resp. not stably but retract k-rational, resp. not retract k-rational) cases of algebraic k-tori of dimension 4. First, we define the weak stably k-equivalence of algebraic k-tori and show that there exist 13 (resp. 128) weak stably k-equivalent classes of algebraic k-tori T of dimension 3 (resp. 4) which are not stably k-rational by computing some cohomological stably birational invariants, e.g. the Brauer-Grothendieck group of X where X is a smooth k-compactification of T, provided by Kunyavskii, Skorobogatov and Tsfasman [KST89]. We make a procedure to compute such stably birational invariants effectively and the computations are done by using the computer algebra system GAP. Second, we define the p-part of the flabby class $[\widehat{T}]^{fl}$ as a $\mathbb{Z}_p[\mathrm{Syl}_p(G)]$-lattice and prove that they are faithful and indecomposable $\mathbb{Z}_p[\mathrm{Syl}_p(G)]$-lattices unless it vanishes for $p = 2$ (resp. $p = 2, 3$) in dimension 3 (resp. 4) via p-adic analysis. The $\mathbb{Z}_p$-ranks of them are also given. Third, we give a necessary and sufficient condition for which two not stably k-rational algebraic k-tori T and T' of dimensions 3 (resp. 4) are stably birationally k-equivalent in terms of the splitting fields and the weak stably k-equivalent classes of T and T'. In particular, the splitting fields of them should coincide if $\widehat{T}$ and $\widehat{T}'$ are indecomposable. Forth, for each 7 cases of not stably but retract k-rational algebraic k-tori of dimension 4, we find an algebraic k-torus T' of dimension 4 which satisfies that $T \times_k T'$ is stably k-rational. Finally, we give a criteria to determine whether two algebraic k-tori T and T' of general dimensions are stably birationally k-equivalent when T (resp. T') is stably birationally k-equivalent to some algebraic k-torus T'' of dimension up to 4. This is a joint work with Akinari Hoshi.

Masahiko Yoshinaga (The University of Osaka)

Varchenko-Gelfand algebra of hyperplane arrangements

The algebra of R-valued functions (where R is a PID) of the set of chambers of a real hyperplane arrangement is called the Varchenko-Gelfand algebra. This algebra has a natural filtration defined by the Heaviside functions. The associated graded algebra is known to be related to the Orlik-Solomon algebra (also the cohomology ring of the complexified complement), in particular, if the characteristic of R is 2, they are isomorphic. In this talk, we discuss the case char R is not equal to 2. We conjecture that one can recover the oriented matroid (the real stratified structure) from the graded VG algebra and present several partial results. This is a joint work with Yukino Yagi (arXiv:2509.19905).

Osamu Iyama (The University of Tokyo)

Cohen-Macaulay representations of Gorenstein rings via Tilting theory

Cohen-Macaulay modules over a Gorenstein ring provide a canonical enhancement of the singularity category. Tilting theory offers a modern framework for studying Cohen-Macaulay representation theory. In this talk, based on joint works with Buchweitz, Yamaura, Kimura, and Ueyama, we present a detailed study of (Artin-Schelter) Gorenstein algebras A of dimension one. We show that the generically projective $\mathbb{Z}$-graded singularity category of A admits a silting object if and only if the degree zero part A_0 has finite global dimension. Moreover, it admits a tilting object if and only if either A is regular or the average Gorenstein parameter g of A is non-positive. We also give explicit constructions of silting objects, and illustrate the results with examples including noncommutative quadrics and tiled orders.

Yu Yasufuku (Waseda University)

GCD of n-variable polynomials and paucity of quasi-integral points

Let $f(x_1, \ldots, x_n), g(x_1, \ldots, x_n)$ be polynomials with coefficients in a number field k. We prove an upper bound of $\mathrm{GCD}(f(P), g(P))$ for $P \in \mathbb{A}^n(k)$. The inequality we obtain is a weaker version of those implied by Vojta's conjecture, which is a deep conjecture in Diophantine geometry. The proof involves Schmidt subspace theorem in the framework of Ru–Vojta theory, as well as algebro-geometric arguments such as intersection theory. We apply the GCD inequality to show paucity of quasi-integral points on some open affine varieties.

Akio Tamagawa (Kyoto University)

Local systems on varieties and their restriction to points in the arithmetico-geometric setting

A local system on a variety is interpreted as a linear representation of the fundamental group of the variety. In the algerbro-geometric setting (over an algebraically closed field), the restriction of a local system to a point is trivial, while in the arithmetico-geometric setting (over an "arithmetic" field), it is highly nontrivial and sometimes even strongly controls the behavior of the original local system. In this talk, I will give a survey (entirely for non-experts) of part of my long-year joint work with Anna Cadoret on this issue.

Tatsuro Kawakami (The University of Tokyo)

Inversion of adjunction for higher rational singularities

The notion of k-rational singularities, introduced by Friedman and Laza, is a generalization of the classical concept of rational singularities. They introduced it motivated by their study of the unobstructedness of deformation theory for singular complex Calabi-Yau varieties. In this talk, we show that an inversion of adjunction holds for k-rational singularities. This is joint work with Jakub Witaszek.

Shunsuke Takagi (The University of Tokyo)

Quasi-F-split and log canonical singularities

Quasi-F-split singularities, introduced by Yobuko, generalize F-split (or F-pure) singularities and have recently been studied intensively in the context of birational geometry. In this talk, we discuss their relationship with log canonical singularities. This talk is based on joint work with Kenta Sato and Shou Yoshikawa.

Akira Ishii (Nagoya University)

Derived McKay correspondence for real reflection groups of rank three

We describe the derived McKay correspondence for real reflection groups of rank 3 in terms of a maximal resolution of the logarithmic pair consisting of the quotient variety and the discriminant divisor with coeficient 1/2. As an application, we verify a conjecture by Polishchuk and Van den Bergh on the existence of a certain semiorthgonal decomposition of the equivariant derived category for any real reflection group of rank 3. This is a joint work with Shu Nimura.

Kento Fujita (The University of Osaka)

Another view on smooth prime Fano threefolds of degree 22 with infinite automorphism groups

All smooth Fano threefolds with infinite automorphism groups are understood due to Prokhorov, Kuznetsov and Shramov by use of deep studies of their Hilbert scheme of lines. I will present as our joint work with Adrien Dubouloz and Takashi Kishimoto an alternative and self-contained proof of it, allowing us to use several properties on the smooth quintic del Pezzo threefold. Moreover, I would like to explain an interesting elementary link joining prime Fano threefolds of degree 22 with Fano threefolds of No. 2.21 in Mori-Mukai's list.

Yoshinori Namikawa (Kyoto University)

Symplectic singularities and Kaledin's conjecture

Symplectic singularities play an important role in algebraic geometry and geometric representation theory. All known examples of such singularities show up with natural C^*-action. About 20 years ago, Kaledin conjectured that a symplectic singularity is always conical; more precisely, it admits a conical C^*-action where the symplectic form is homogeneous. Recently we proved Kaledin's conjecture conditionally, but in a substantially stronger form. The idea is to use Donaldson-Sun theory in complex differential geometry to connect with the theory of Poisson deformations of symplectic varieties. This is a joint work with Y. Odaka.

Organizers: Kenta Hashizume, Akinari Hoshi, Hideo Kojima, Takuzo Okada, Kazuhiko Yamaki.