List of Problems

1. **Plane Curves C**
   The ground field k of the discussion is assumed to be an algebraically closed field of characteristic zero if it is not mentioned otherwise. Let d be the degree of C, where we assume $d \geq 3$. Find the Galois points P for C and their Galois groups of the following cases.
   (a) Galois points and Galois groups
      (i) Let F_d be the Fermat curve $d \geq 5$. Suppose that $d - 1$ is not a prime number. Then, what is the Galois group at the flexes ? For example, if $d = 7$, then is it true G_P is isomorphic to D_6 ? (cf. [20], [27])
      (ii) Elliptic curves with singularities. The case of rational curves has been determined [33].
      (iii) Let C be a singular plane quartic and $P \in \mathbb{P}^2 \setminus C$. Then consider when P is a Galois point (cf. [14]). Find the characterization of the curve with the maximal number of Galois points.
      (iv) Let C be a singular plane quintic and $P \in C$. Then consider when P is a Galois point.(cf. [26])
      (v) Study Galois points and their Galois group for quintic singular curves. In particular at singular points. Do there exist a quintic curve with two Galois points such that the gorup are cyclic group and Klein's four group respectively ?
      (vi) Find the estimate of the number of Galois points whose Galois group is isomorphic to an alternating group.
      (vii) Let C be the sigular plane sextic with 10 double points. Find inner and outer Galois points for it.
      (viii) Suppose $P \in C$ is a singular Galois point. Then, what can we say about the property of the singularity ?
      (ix) If the Galois group is abelian, then what can we say about the structure of the gorup ?

   (b) The element of G induces a biratinal transformation on C over $\mathbb{P}^1$. When is it extendable to a birational transformation of $\mathbb{P}^2$? (cf. [17] and [34])

   (c) When P is not a Galois point, we consider the Galois closure of the field extension. (cf. [27])
      (i) For a given smooth curve C, find the Galois group and the Galois closure curve for C, in particular the genus of the Galois closure curve.
      (ii) If P is close to P', then are the Galois closure curves not isomorphic to each other ? (cf. [22] and [30])
      (iii) Consider the total space of Galois closure curves for C (cf. [23]).

2. **Plane Curve over k with positive charcteristic**

It may be quite interesting to study the above problems in the case where the ground field k has positive characteristic, in particular when k is a finite field. Recently, it has become clear that there are many different results from characteristic zero case and there have been a big progress of the research on these topics. (cf. [5],[6],[7],[8],[9] [12])

(a) Study the same problems as in the case where the ground field is an algebraically closed field with positive characteristic, i.e., find Galois points, Galois groups .....

(b) Study the relations between Galois points and rational points in the case where k is a finite field. ([11])

(c) Do there any relations between Galois points and rational points when k is a finite field ?

(d) How are the distributions of Galois points ?

3. **Space Curves C in $\mathbb{P}^3$**

(a) Find Galois lines for C, or in particular, the estimate of the number of Galois lines. (cf. [31])

(b) Study the Galois lines l and Galois group for normal quartic space curve C in the case where $l \cap C \neq \emptyset$ (cf. [4]).

4. **Space Curves C in $\mathbb{P}^n$**, $n \geq 3$

(a) Find Galois subspaces for C, or in particular, the estimate of the number of Galois subspaces.

(b) Study the Galois subspaces L and Galois group for rational normal curve C. In particular, find the Grassmann variety of the subspaces.

(c) Study the Galois group of the Galois closure of the extension when the subspace is not Galois. ([21])

5. **Projective Surfaces** S

(a) Find projective embedding of S, in which there exists Galois subspaces.

(b) When S is embedded as a hypersurface, study the same problems as in the case of curves, i.e., find Galois points, Galois groups .....

(c) Find the Galois points for smooth hypersurfaces. (cf. [28] and [29])

(d) Find the number of Galois points for normal hypersurfaces of degree ≥ 5 (cf. [25])

(e) For abelian, hyperelliptic, $K3$ and Enriques surfaces consider the Galois embeddings, the Galois subspaces and the Galois groups. (cf. [32])

(f) Similarly as in the case of curves, if P is not a Galois point, then consider the Galois closure of the extension and find the relative minimal model of the Galois closure. (cf. [24])

6. **The most general case**

Ler V be a smooth variety with very ample divisor D. Embedd V into $\mathbb{P}^N$ by the complete linear system $|D|$. Then consider a projection by linear subvariety in $\mathbb{P}^N$. If the projection induces a Galois extension of function fields, the embedding is said to be Galois embedding. (cf. [32])

(a) $\{ W \in \mathbb{G}(N-n-1, N) \mid G_W \cong S_d \}$. In particular, is it true that the codimension of the complement of the set is at least two ?

(b) Suppose that $\dim Lin(V) = 0$, W is close to W' (in the Grassmanian) and $W \neq W'$. Let K_W be the Galois closure of the extension determined by the projection. Then is it true that K_W is not isomorphic to $K_{W'}$?

(c) For an embedding (V, D) find the structure of Galois group G_W for each $W \in \mathbb{G}(N-n-1, N)$.

(d) Find all the Galois subspaces for one Galois embedding. In particular, find the rule of arrangements of Galois subspaces.

(e) Consider the similar subject in the case where $f(V) \cap W \neq \emptyset$.

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